EE3303 ELECTRICAL MACHINES - I L T P C 3 0 0 3

UNIT I ELECTROMECHANICAL ENERGY CONVERSION 9

Fundamentals of Magnetic circuits- Statically and dynamically induced EMF - Principle of electromechanical energy conversion forces and torque in magnetic field systems- energy balance in magnetic circuits- magnetic force- co-energy in singly excited and multi excited magnetic field system mmf of distributed windings – Winding Inductances-, magnetic fields in rotating machines- magnetic saturation and leakage fluxes. Introduction to Indian Standard Specifications (ISS) - Role and significance in testing.

UNIT II DC GENERATORS 9

Principle of operation, constructional details, armature windings and its types, EMF equation, wave shape of induced emf, armature reaction, demagnetizing and cross magnetizing Ampere turns, compensating winding, commutation, methods of improving commutation, interpoles, OCC and load characteristics of different types of DC Generators. Parallel operation of DC Generators, equalizing connections- applications of DC Generators.

UNIT III DC MOTORS 9

Principle of operation, significance of back emf, torque equations and power developed by armature, speed control of DC motors, starting methods of DC motors, load characteristics of DC motors, losses and efficiency in DC machine, condition for maximum efficiency. Testing of DC Machines: Brake test, Swinburne’s test, Hopkinson's test, Field test, Retardation test, Separation of core losses-applications of DC motors.

UNIT IV SINGLE PHASE TRANSFORMER 9

Construction and principle of operation, equivalent circuit, phasor diagrams, testing - polarity test, open circuit and short circuit tests, voltage regulation, losses and efficiency, all day efficiency, back-toback test, separation of core losses, parallel operation of single-phase transformers, applications of single-phase transformer.

UNIT V AUTOTRANSFORMER AND THREE PHASE TRANSFORMER 9

Construction and working of auto transformer, comparison with two winding transformers, applications of autotransformer. Three Phase Transformer- Construction, types of connections and their comparative features, Scott connection, applications of Scott connection.

TOTAL: 30+15=45

PERIODS TEXT BOOKS

1. I. J. Nagrath and D. P. Kothari, “Electric Machines”, McGraw Hill Education, 5th Edition, 2017.
2. P. S. Bimbhra, “Electric Machinery”, Khanna Publishers, 2nd Edition, 2021.

REFERENCES

1. A. E. Fitzgerald and C. Kingsley, "Electric Machinery”, New York, McGraw Hill Education, 6th Edition 2017.
2. A. E. Clayton and N. N. Hancock, “Performance and design of DC machines”, CBS Publishers, 2018.
3. M. G. Say, “Performance and design of AC machines”, CBS Publishers, First Edition 2008.
4. Sahdev S. K. “Electrical Machines”, Cambridge University Press, 2018.

UNIT - I

ELECTRO MECHANICAL ENERGY CONVERSION

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ELECRICAL MACHINE TYPES

There are three basic rotating electric machine types, namely

1. The DC machine
2. The polyphase synchronous machine (ac)

and 3. the polyphase induction machine (ac)

Three materials are mainly used in machine manufacture; steel to conduct magnetic flux, copper to conduct electric current and insulation to insulate the voltage induced in conductors confining current to them.

Allelectric machines comprise of two parts; the cylinderical rotating member called the rotor and the annular stationary member called the stator with the intervening air-gap. The rotor has an axial shaft which is carried on bearings at each end located in end covers bolted to the stator. The shaft extends out of the end cover usually at one end and is coupled to either the prime mover or the load.

In both dc and synchronous machines, the main field is created by field poles excited with direct current. The windings on the field poles is called the field winding. the relative motion of the field past a second winding located in the other member induces emf in it. the winding interchanges current with the external electric system depending upon the circuit conditions. It is this winding, called the armature winding, which handles the load power of the machine, while the field winding consumes a small percentage of the rated load power.

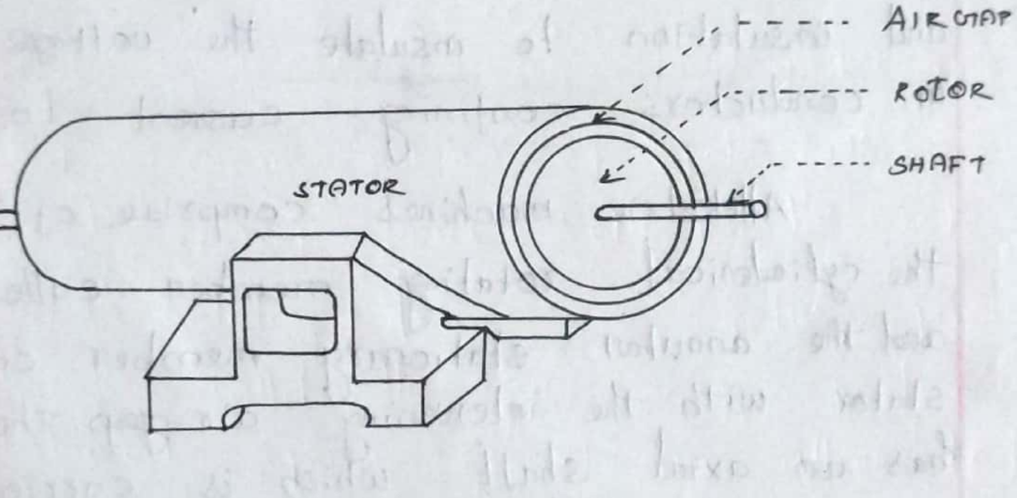

FIG. AN ELECTRIC MACHINE

DC MACHINE

In a DC machine the field poles are on the stator while the rotor is the armature as shown in fig. As the armature rotates, alternating emf and current induced in the armature winding are rectified to DC form by a rotating mechanical switch called the commutator which is tapped by means of stationary carbon brushes. The armature when carrying current produces stationary poles which interact with the field poles to produce the electromagnetic torque.

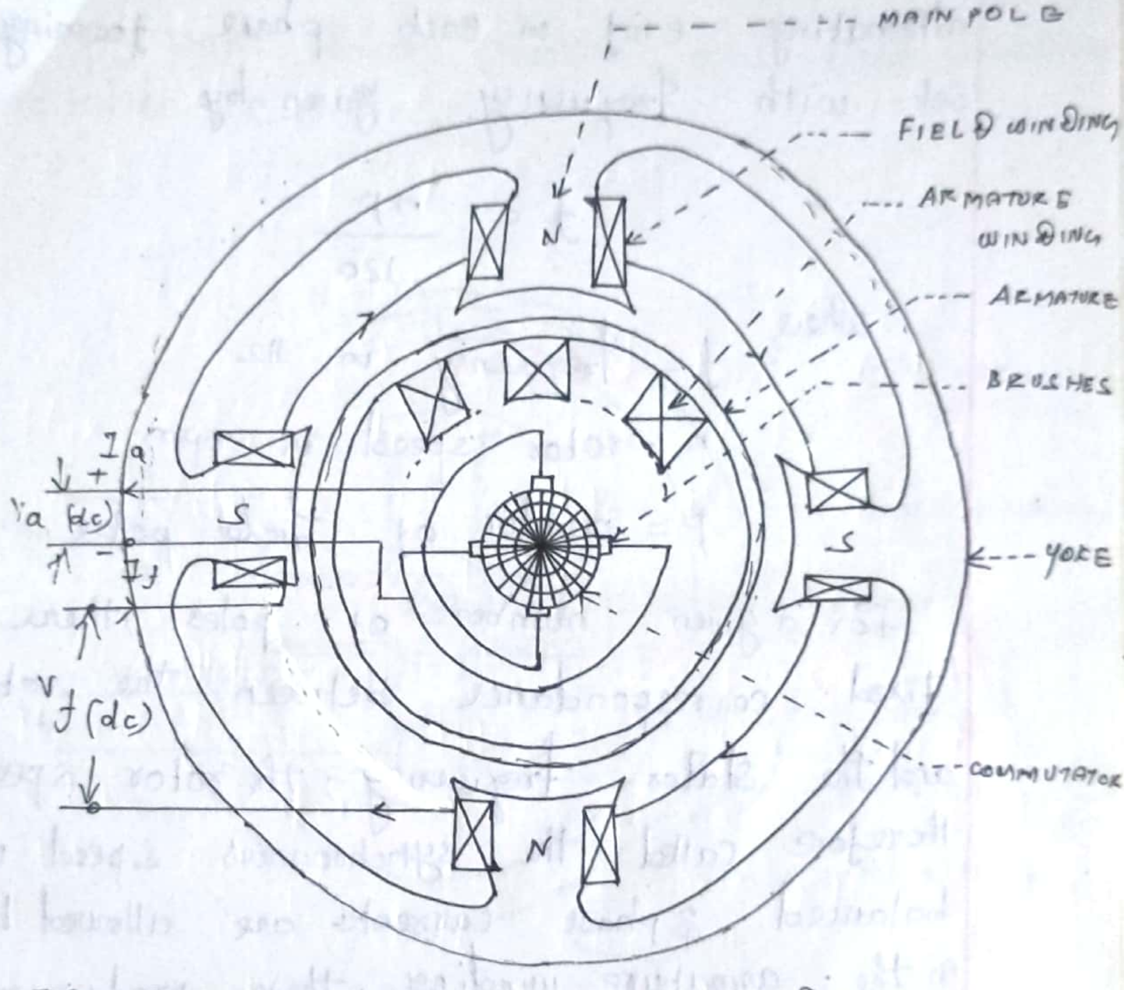

FIG. CROSS SECTIONAL VIEW OF DC MACHINE

SYNCHRONOUS MACHINE:

In a synchronous machine the field poles could be either on the stator or rotor, but in all practical machines the rotor carries the field poles as shown in fig. The field poles are excited by DC current. The stator forms the armature carrying a 3-phase winding wound for the same number of poles as the rotor. All the three phases have identical windings with the same angular displacement between any pair of phases. when the rotor rotates, it produces alternating emf in each phase forming a balanced set with frequency given by

$$f = \frac{nP}{120}$$

where

f = frequency in Hz

n = rotor speed in rpm

P = number of field poles.

For a given number of poles, there is a fixed correspondance between the rotor speed and the stator frequency; the rotor speed is therefore called the synchronous speed. when balanced 3 phase currents are allowed to flow in the armature winding, these produce a synchrono-usly rotating field, stationary with respect to the rotor field as result of which the machine produces torque of electromagnetic origin. The synchronous motor is, however, non self starting.

In both dc and synchronous machine the power handling capacity is determined by the voltage and current of the armature winding, while the field is excited from low power dc. Thus these machine types are doubly excited. Quite different from these, an induction machine is singly excited from 3-phase mains on the stator side. The stator must therefore carry both load current and field producing excitation current.

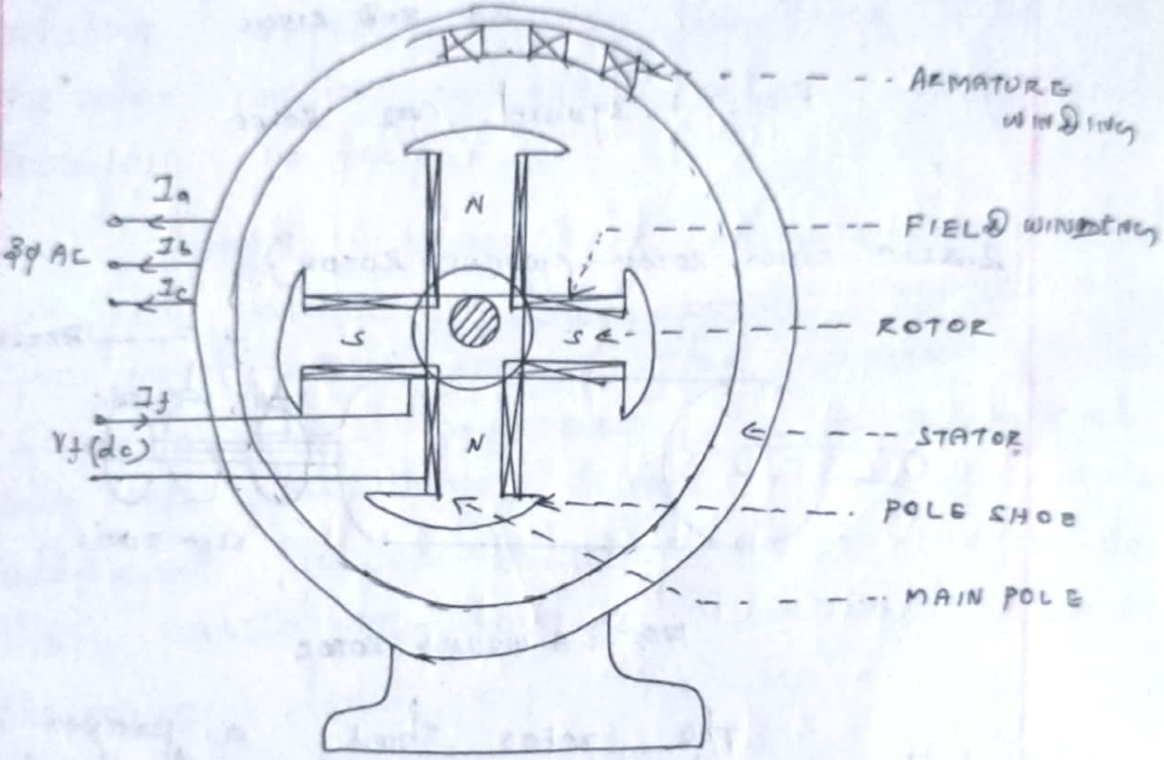

FIG. CROSS SECTIONAL VIEW OF SYNCH-RONOUS MACHINE.

INDUCTION MACHINE:

TWO types of rotor constructions are employed which distinguish the type of induction motor.

1. SQUIRREL-CAGE ROTOR: Here the rotor has copper bars embedded in slots which are

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short circuited at each end. It is rugged economical construction but develop low starting torque.

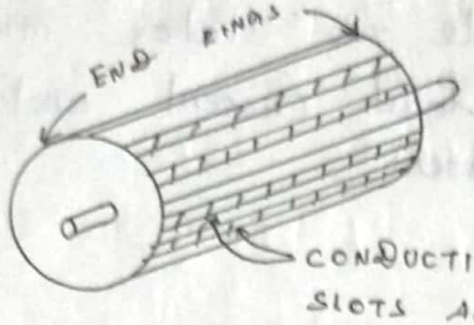

Fig: A squirrel cage Rotor.

2. SLIP RING ROTOR (WOUND ROTOR)

FIG: A WOUND ROTOR

The rotor has a proper 3 phase winding with three leads brought out through slip rings and brushes. These leads are normally short circuited when the motor is running. Resistances are introduced in the rotor circuit via the slip rings at the time of starting to improve the starting torque.

The rotating field created by the stator winding moves past the shorted rotor conductors inducing current in it. These induced currents

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produce their own field which rotates at the same speed with respect to the stator as the stator - produced field. Torque is developed by the interaction of these two relatively stationary fields. The rotor runs at a speed close to synchronous but always slightly lower than it. At the synchronous speed no torque can be developed as zero relative speed between the stator field and the rotor implies no induced rotor currents and therefore no torque.

single phase ac motors are employed for low voltage, low power applications - fractional kW motors. They operate on the same basic principles as the 3-phase motor, but the pulsating single phase field produce additional losses, reducing motor torque and the pulsating torque component increases the noise level of the motor.

An induction machine connected to the mains when driven at supersynchronous speed behaves as a generator feeding power into the electric system. It is used in small hydroelectric stations and wind and aerospace applications.

MOTOR CONTROL :

There is great diversity and variety in the components and systems used to control rotating machines. The purpose of a motor control may be as simple as start/stop or the control of one or more of the motor output parameters ie shaft speed, angular position, acceleration, shaft torque and mechanical power output. with the rapid development of solid state power devices, integrated circuits and cheap computer modules, the range, quality and accuracy of electronic motor control has become almost infinite.

ECONOMIC AND OTHER CONSIDERATION.

Economics is an important consideration in the choice of electric machines. The trade off between the initial capital cost and the operating and maintenance cost must be taken into account in this choice.

MAGNETIC CIRCUITS AND INDUCTANCE

MAGNETIC CIRCUIT:

It may be defined as the route or path which is followed by magnetic flux. the laws of magnetic ckt are quite similar to those of the electric ckt.

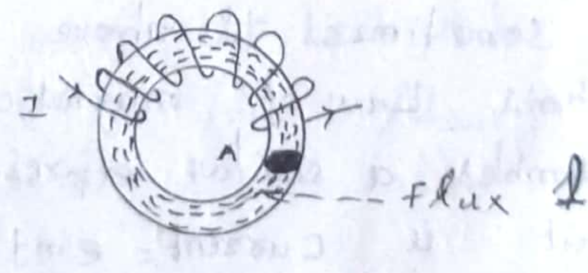
The exact description of the magnetic field is given by the Maxwell's equations and the constitutive relationship of the medium in which the field is established

The ampere's law is reproduced as follows.

$$\int_s \vec{J} \cdot \vec{ds} = \oint \vec{H} \cdot \vec{dl}$$

where in $\vec{J}$ = conduction current density
$\vec{H}$ = magnetic field intensity
S = the surface enclosed by the closed path of length l.
$\vec{ds}$ = differential surface
$\vec{dl}$ = differential length.

consider a solenoid or a toroidal iron ring having a magnetic path of l metre, area of cross section A m^2 and a coil of N turns carrying I amperes wound anywhere on it.



The field strength inside the solenoid is $H = \frac{NI}{l}$ AT/m

Now $B = \mu_0 \mu_r H$

$\therefore B = \frac{\mu_0 \mu_r NI}{l}$ wb/m^2

Total flux produced

$$\phi = B \times A = \frac{\mu_0 \mu_r A N I}{l} \text{ wb}$$

$$\phi = \frac{NI}{l/\mu_0 \mu_r A} = \frac{NI}{S}$$

The numerator NI which produces magnetisation in the magnetic circuit is known as magnetomotive force. obviously, its unit is ampere - turns.

It is analogous to e.m.f in electric circuit.

The denominator $\frac{l}{\mu A}$ or $\frac{l}{\mu_0 \mu_r A}$ is called the reluctance of the circuit and is analogous to resistance in electric circuit.

$$\therefore \text{flux} = \frac{\text{mmf}}{\text{reluctance}}$$

Sometimes, the above equation is called the ohm's Law of magnetic circuit because it resembles a similar expression in electric circuit ie current = $\frac{\text{emf}}{\text{resistance}}$

DEFINITIONS:

1. Magnetomotive force (mmf) It drives or tends to drive flux through a magnetic

circuit and corresponds to electromotive force in an electric circuit. It is given by the product NI.

MMF is equal to the work done in joules in carrying a unit magnetic pole once through the entire magnetic circuit. It is measured in ampere-turns.

2. Ampere-turns : It is the unit of magneto motive force.

3. Reluctance :

It is the name given to that property of a material which opposes the creations of magnetic flux in it. Its unit is AT/wb.

$$\text{Reluctance} = \frac{l}{\mu_0 \mu_r A} \text{ or } \frac{l}{\mu A}$$

$$\text{Resistance} = \rho \frac{l}{A} = \frac{l}{\sigma A}$$

In other words, reluctance of a magnetic circuit is the number of ampere-turns required per weber of magnetic flux in the circuit. Since 1AT/wb = 1/henry, hence unit of reluctance is also reciprocal henry.

4. Flux : It is equal to the total number of lines of induction existing in a magnetic circuit and is analogous to current in an electric circuit. It is measure in webers.

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5. permeance : It is reciprocal of reluctance and implies the ease or readiness with which magnetic flux is developed. It is analogous to conductance in electric circuits. It is measured in terms of wb/AT or henry.

6. Reluctivity : It is specific reluctance and corresponds to resistivity which is specific resistance.

CORE WITH AIR GAP:

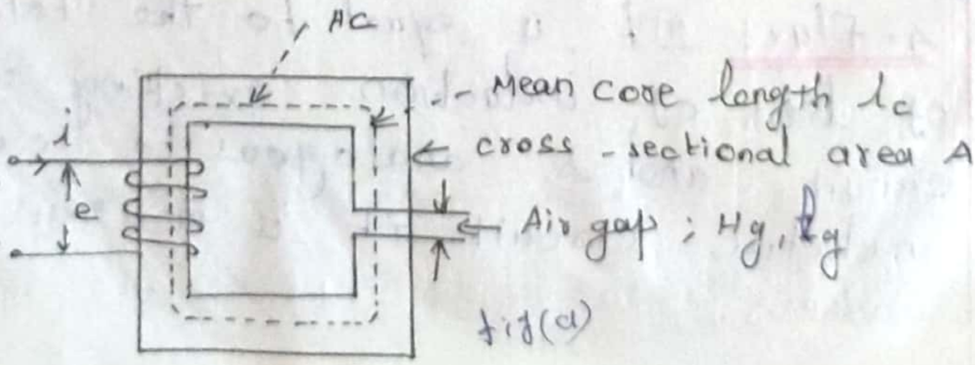
A typical magnetic circuit with an air gap is shown in fig.

Actually as will soon be seen, that the flux in the gap fringes out so that the gap flux density is somewhat less than that of the core. Further, let the core permeability μ_c be regarded as constant.

The mmf Ni is

$$Ni = H_c l_c + H_g l_g \quad \text{--- (1)}$$

$$Ni = \frac{B_c}{\mu_c} l_c + \frac{B_g\, l_g}{\mu_g \text{ or } \mu_0} \quad \text{--- (2)}$$



fig(a)

Assuming that all the core flux passes straight down the air-gap

$$B_g = B_c$$

$$\therefore \varphi = B_c A = B_g A \quad \text{--- (9)}$$

sub (9) in (8)

$$Ni = \varphi\left[\frac{l_c}{\mu_c A}\right] + \varphi\left[\frac{l_g}{\mu_0 A}\right]$$

$$H = \varphi \underset{\text{core}}{S_c} + \varphi \underset{\text{air gap}}{S_g}$$

$$= \varphi (S_c + S_g)$$

$$\varphi = \frac{H}{S_c + S_g} = \frac{H/S_g}{1 + S_c/S_g}$$

$$\frac{S_c}{S_g} = \frac{\mu_0 l_c}{\mu_c l_g} << 1$$

$$\therefore l_c >> l_g$$

$$\varphi \approx H/S_g$$

LEAKAGE flux:

Leakage flux is the flux which follows a path not intended for it.

The flux in the air gap is known as the useful flux.

magnetic leakage can be minimised by placing the exciting coils or windings as closely as possible to the air-gap or to the points in the magnetic circuit where flux is to be utilised for useful purposes.

FRINGING:

At an air-gap in a magnetic core, the flux fringes out into neighbouring

FRINGING:

At an air gap in a magnetic core, the flux fringes out into neighbouring air paths as shown in fig. The result is nonuniform flux density in the air gap. enlargement of the effective air gap area and a decrease in the average gap flux density. the fringing effect also disturbs the core flux pattern.

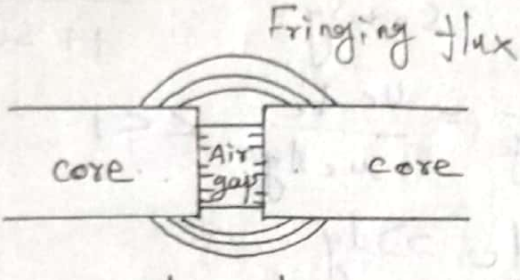


Fig. Flux fringing at air gap.

The effect of fringing increases with the air gap length.

For the example of the core with the air gap previously presented, the gap reluctance would now be given by

$$S_g = \frac{l_g}{\mu_0 A_g}$$

which will be less than the previous value as $A_g > A$.

STACKING FACTOR:

Magnetic cores are made up of thin, tightly insulated laminations to reduce power less in core due to the eddy current phenomenon

As a result, the net cross sectional area of the core occupied by the magnetic material is less than its grass cross-section; their ration (less than unity) being known as the stacking factor. It may vary from 0.5 – 0.95.

EXAMPLE The magnetic circuit of fig (a) has dimensions $A_c = 4 \times 4\ cm^2$, $l_g = 0.06\ cm$, $l_c = 40\ cm$; $N = 600$ turns. Assume the value of $\mu_r = 6000$ for iron. Find the exciting current for $B_c = 1.2\ T$ and the corresponding flux and flux linkages.

soln: Ampere turns for the circuit are given by,

$$Ni = \frac{B_c}{\mu_0 \mu_r} l_c + \frac{B_g}{\mu_0} l_g$$

Neglecting fringing, $A_c = A_g$ therefore $B_c = B_g$

$$i = \frac{B_c}{\mu_0 N}\left(\frac{l_c}{\mu_r} + l_g\right)$$

$$= \frac{1.2}{4\pi \times 10^{-7} \times 600}\left(\frac{40}{6000} + 0.06\right) \times 10^{-2}$$

$$\boxed{i = 1.061A}$$

The reader should note that the reluctance of the iron path of 40 cm is only $\left(\frac{2/3}{6}\right) = 0.11$ of the reluctance of the 0.06 cm air-gap.

$$\phi = B_c A_c = 1.2 \times 16 \times 10^{-4} = 19.2 \times 10^{-4}\ Wb$$

Flux linkages $\lambda = N\phi = 600 \times 19.2 \times 10^{-4} = 1.152$ Wb-turns

If fringing is to be taken into account, one gap length is added to each dimensions of the air-gap constituting the area, then

$$A_g = (4+0.06)(4+0.06) = 16.484 cm^2$$

$A_g > A_c$

$$B_g = \frac{\phi}{A_g}$$

$$B_g = \frac{19.2 \times 10^{-4}}{16.484 \times 10^{-4}} = 1.165 T$$

$$i = \frac{1}{\mu_0 N}\left(\frac{B_c l_c}{\mu_r} + B_g l_g\right)$$

$$= \frac{1}{4\pi \times 10^{-7} \times 600}\left[\frac{1.2 \times 40 \times 10^{-2}}{6000} + 1.165 \times 0.06 \times 10^{-2}\right]$$

$$\boxed{i = 1.0832 A}$$

EXAMPLE A wrough iron bar 30 cm long and 2 cm in diameter is bent into a circular shape as shown in fig. It is then wound with 600 turns of wire. Calculate the current required to produce a flux of 0.5 mWb in the magnetic ckt in the following cases.

(i) no air gap

(ii) with an air gap of 1 mm

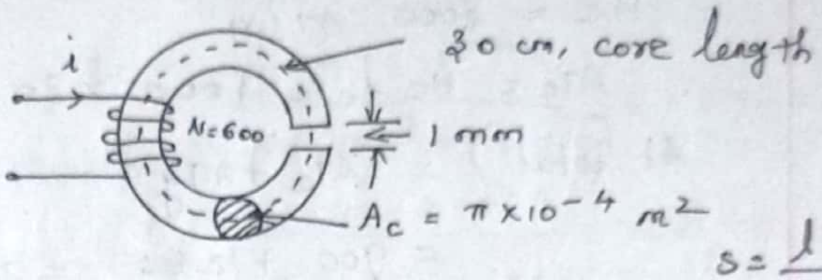
μ_r (iron) = 4000 (assumed constant)

1 mm

and

(iii) with an air gap of 1 mm : assume the following data for the magnetisation of iron.

H in AT/m	2500	3000	3500
B in T	1.55	1.59	1.6



Soln:

$S = \frac{l}{\mu A}$

(i) No air-gap $\quad S_c = \frac{l_c}{\mu_0 \mu_r A} \quad mmf = \phi \cdot S$

$$S_c = \frac{30\times10^{-2}}{4000 \times 4\pi\times10^{-7} \times \pi\times10^{-4}} = 1.9\times10^{5}$$

$$Ni = \phi S_c$$

$$\text{or } i = \phi S_c / N = \frac{0.5 \times10^{-3} \times 1.9\times10^{5}}{600}$$

$$= 0.158\ A$$

(ii) Air gap = 1 mm $\quad \mu_r(\text{iron}) = 4000$

$$S_c = 1.9\times10^{5} \qquad S_g = \frac{l_g}{\mu_0 A}$$

$$S_g = \frac{1\times10^{-3}}{4\pi\times10^{-7} \times\pi\times10^{-4}} = 25.33\times10^{5}$$

$$S(\text{total}) = S_c + S_g = 27.1\times10^{5}$$

$$\therefore i = \frac{0.5\times10^{-3} \times 27.1\times10^{5}}{600} = 2.258\ A$$

$\left[\because i = \frac{\phi S}{N}\right]$

(iii) Air-gap = 1 mm; B-H data as given

$B = \phi/A$

$$B_c = B_g = \frac{0.5\times10^{-3}}{\pi\times10^{-4}} = 1.59\ T \text{ (fringing neglected)}$$

$B = \mu H$

$$H_g = \frac{B_g}{\mu_0} = \frac{1.59}{4\pi\times10^{-7}}$$

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mmf

$$AT_g = H_g l_g = \frac{1.59 \times 1 \times 10^{-3}}{4\pi \times 10^{-7}} = 1265$$

From the given magnetization data

at $(B_c = 1.59T)$

$H_c = 3000$ AT/m

$AT_c = H_c l_c = 3000 \times 30 \times 10^{-2} = 900$

$At(\text{total}) = AT_c + AT_g$

$= 900 + 1265 = 2165$

$i = \frac{2165}{600}$ mmf

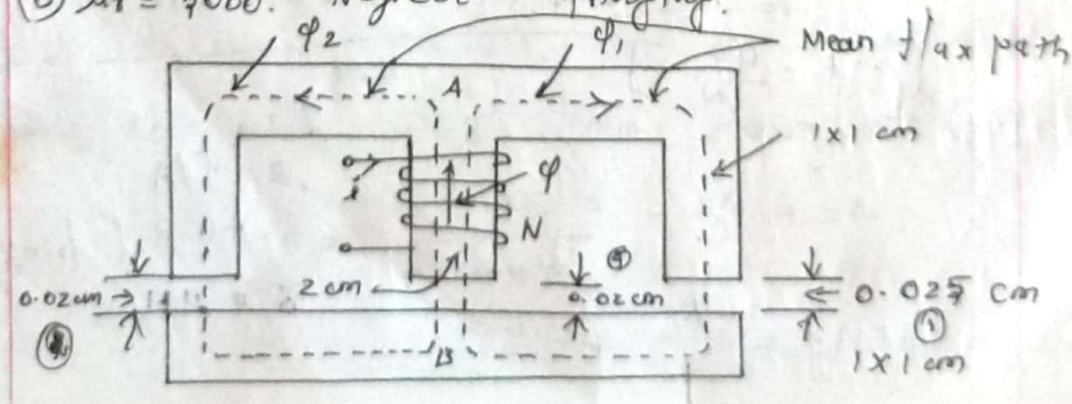
$$\boxed{i = 3.61A}$$

Example: The magnetic ckt of fig. has cast steel core with dimensions as shown.

Mean length from A to B through either outer limb $= 0.5$ m

Mean length from A to B through the central limb $= 0.2$ m.

In the magnetic cct shown it is requiried to establish a flux of 0.75 m wb in the air gap of the central limb. Determine the mmf of the exciting coil if for the core material (a) $\mu_r = \alpha$ (b) $\mu_r = 5000$. neglect fringing.



Soln:

(a) $\mu_r = \infty$ ie there are no mmf drops in the magnetic core. Various gap reluctances are:

$$\frac{l}{\mu A} = S$$

$$S_{g_1} = \frac{0.025 \times 10^{-2}}{4\pi \times 10^{-7} \times 1 \times 10^{-4}} = 1.99 \times 10^{6}$$

$$S_{g_2} = \frac{0.02 \times 10^{-2}}{4\pi \times 10^{-7} \times 1 \times 10^{-4}} = 1.592 \times 10^{6}$$

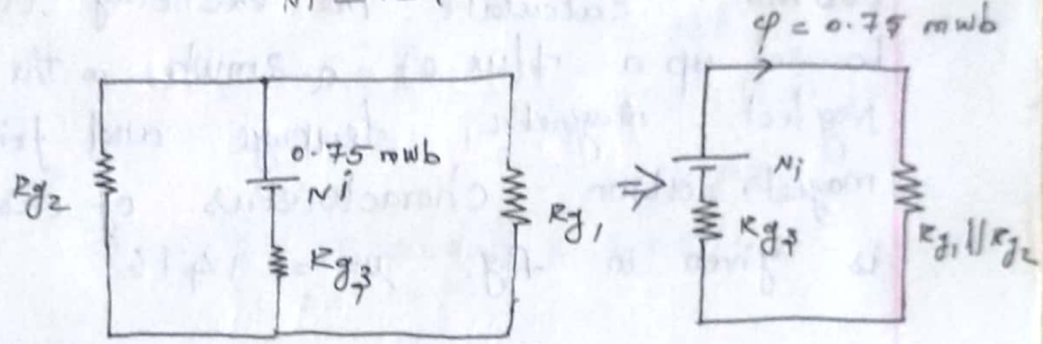
$$S_{g_3} = \frac{0.02 \times 10^{-2}}{4\pi \times 10^{-7} \times 2 \times 10^{-4}} = 0.796 \times 10^{6}$$

$NI = 0.75 \times 10^{-3} (R_{g_3} + R_{g_1} \parallel R_{g_2})$

$= 0.75 \times 10^{-3} (0.796 + 0.844) \times 10^{6}$

$= 1230$ AT

$NI = 1230$ AT.



(b) $\mu_r = 5000$. this means that the reluctance of magnetic core must be taken into consideration. The analogous electric ckt now becomes that various core reluctances can be calculated as follows.

$$S_{c_1} = \frac{0.5}{4\pi \times 10^{-7} \times 5000 \times 1 \times 10^{-4}} = 0.796 \times 10^{6}$$

$$S_{c_2} = S_{c_1} = 0.796 \times 10^{6}$$

$$S_{c_3} = \frac{0.2}{4\pi \times 10^{-7} \times 5000 \times 2 \times 10^{-4}} = 0.159 \times 10^{6}$$

the equivalent reluctance is

$$R_{eq} = (S_{c_1} + S_{g_1}) \parallel (S_{c_2} + S_{g_2}) + S_{c_3} + S_{g_3}$$

$$= \frac{27.86 \times 23.86}{51.72} \times 10^{6} + 0.955 \times 10^{6}$$

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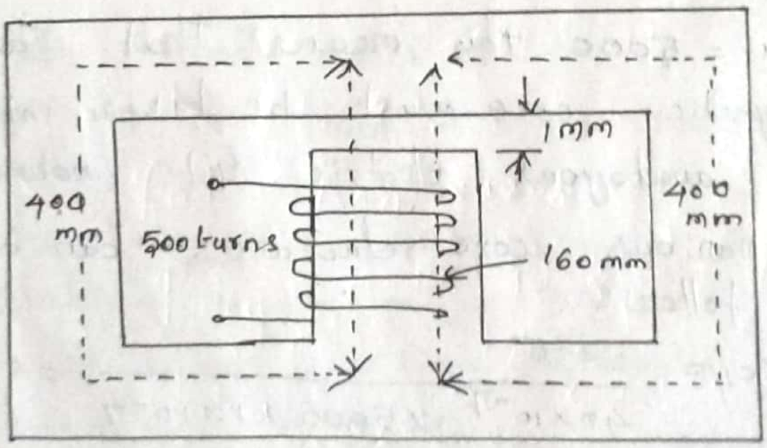
$= 1.955 \times 10^{6}$

Now, $Ni = \phi R_{eq}$

$= 0.75 \times 10^{-3} \times 1.955 \times 10^{6}$

$= 1466 AT.$

Example: The magnetic ckt of fig has cast steel core. The cross sectional area of the central limb is 800 mm^2 and that of each outer limb is 600 mm^2. calculate the exciting current needed to set up a flux of 0.8 mwb in the air gap. Neglect magnetic leakage and fringing. The magnetization characteristics of cast iron steel is given in fig. $\mu_r = 1416$



Soln:

Air gap

$B = \frac{\phi}{A}$ $\quad mm^2 = (10^{-3})^2 = 10^{-6}$

$B_g = \frac{0.8}{800} \times \frac{10^{-3}}{10^{-6}} = 1 T$ (or) Wb/m^2 $\quad \phi = \frac{Ni}{S}$ $\quad Ni = H_g l_g$

$B = \mu H = \mu_0 \mu_r H$

and $H_g = \frac{1}{4\pi \times 10^{-7}}$ AT/m

mmf, $F_g = \frac{1}{4\pi \times 10^{-7}} \times 1 \times 10^{-3}$

$= 796$ AT

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central limb

$B_c = B_g = 1T$

$H_c = 1000$ AT/m (in cast steel for 1T, $H = 1000$ AT/m)

mmf, $F_c = H_c \times l_c = 1000 \times 160 \times 10^{-3}$

$= 160$ AT

Because of symmetry, flux divides equally between the two outer limbs, so

ϕ (outer limb) $= 0.8/2 = 0.4$ mwb.

$B = \phi/A$

B (outer limb) $= \dfrac{0.4 \times 10^{-3}}{600 \times 10^{-6}} = 0.667$ AT

$B = \mu H$

$H = 0.667$

mmf F(outer limb) $= 375 \times 400 \times 10^{-3}$

$= 150$ AT

F(total) $= 796 + 160 + 150$

$= 1106$ AT.

Exciting current $= 1106/500 = 2.21$ A

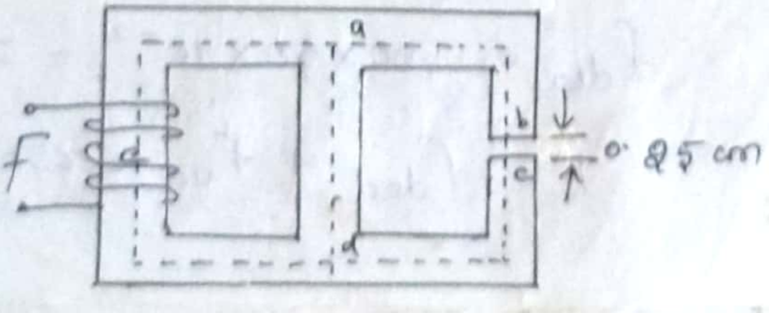
Example 5: The magnetic circuit of fig has a cast steel core whose dimensions are given below.

Length (ab+cd) $= 50$ cm cross sectional area $= 25$ cm^2

length ad $= 20$ cm. cross sectional area $= 12.5$ cm^2

length dea $= 50$ cm cross-sectional area $= 25$ cm^2

Determine the exciting coil mmf required to establish an air-gap flux of 0.75 mwb.



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Soln: Assuming no fringing the flux density in the path abcd will be same, ie

$$B = \frac{\varphi}{A} = \frac{0.75 \times 10^{-3}}{25 \times 10^{-4}} = 0.3T.$$

$$F_{bc} = \frac{B_0}{\mu_0} l_{bc}$$

$$= \frac{0.3 \times 0.25 \times 10^{-3}}{4\pi \times 10^{-7}} = 60 AT.$$

$H_{ab} = H_{cd}$ (for cast steel $B = 0.3T$)

$= 200 AT/m$

$$F_{ab+cd} = 200 \times 50 \times 10^{-2} = 100 AT$$

$$F_{ad} = 60 + 100 = 160 AT$$

$$H_{ad} = \frac{160}{20 \times 10^{-2}} = 800 AT/m$$

$$B_{ad} = 1.04 T$$

$$\varphi_{ad} = 1.04 \times 12.5 \times 10^{-4} = 1.3 mWb$$

$$\varphi_{dea} = 0.75 + 1.3 = 2.05 mWb$$

$$B_{dea} = \frac{2.05 \times 10^{-3}}{25 \times 10^{-4}} = 0.82T$$

$$H_{dea} = 500 AT/m$$

$$F_{dea} = 500 \times 50 \times 10^{-2} = 250 AT$$

$$F = F_{dea} + F_{ad} = 250 + 160 = 410 AT$$

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Example 6 A cast steel ring has a circular cross section of 3 cm in diameter and a mean circumference of 80 cm. A 1 mm air-gap is cut out in the ring which is wound with a coil of 600 turns.

(a) Estimate the current required to establish a flux of 0.75 mwb in the air gap. neglect fringing and Leakage.

(b) what is the flux produced in the air gap if the exciting current is 2A? Neglect fringing and leakage.

Magnetization data :

H (AT/m)	200	400	600	800	1000	1200	1400		
B (T)	0.10	0.32	0.60	0.90	1.08	1.18	1.27		
	1600	1800	2020						
	1.32	1.36	1.40						

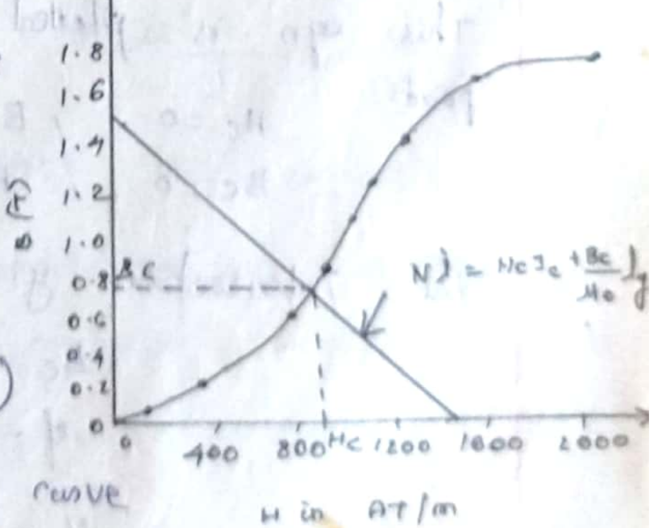
soln:

$$\varphi = 0.75 \times 10^{-3} \text{ wb}$$

$$B_g = \varphi / A = \frac{0.75 \times 10^{-3}}{\pi \times \left(\frac{0.03}{2}\right)^2}$$

$$= 1.067$$

$B_c = B_g$ (no fringing)



Reading from the B-H curve

$H_c = 900$ AT/m

25

$l_c = 0.8\,m$ (air gap length can be neglected)

$AT_c = H_c l_c = 900 \times 0.8 = 720$

$AT_g = \frac{1.06}{4\pi \times 10^{-7}} \times 10^{-3} \quad \frac{B_g l_g}{\mu_0} = 843$

$Ni = AT_c + AT_g = 720 + 843 = 1563$

$\therefore i = \frac{1563}{600} = 2.6\,A$

(b) The excitation is now given and the flux is to be determined from the B-H curve given. The problem must, therefore $\rightarrow B_g = B_c$ $\quad H_c = \frac{B_c}{\mu_0}$

$$Ni = \frac{B_c}{\mu_0} l_g + H_c l_c \; ; \; (B_g = B_c) \quad \text{—①}$$

The intersection of the two for a given Ni will yield the solution. For this problem

$$Ni = 600 \times 2 = 1200\,AT.$$

Substituting various values in eqn(1)

$$1200 = \frac{B_c}{4\pi \times 10^{-7}} \times 10^{-3} + 0.8 H_c$$

This eqn is plotted in fig , by locating the points.

$H_c = 0$, $B_c = 1.5$

$B_c = 0$, $H_c = 1500$

The intersection gives the result

$B_c = 0.78\,T$

$\phi = B_c A = 0.78 \times \frac{\pi}{4} (0.03)^2$

$\phi = 0.55\,mWb$

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INDUCTANCE:

The equation may be written as

$$e = N \cdot \frac{d\phi}{dt} = N \cdot \frac{d\phi}{di} \frac{di}{dt} = L \cdot \frac{di}{dt} \quad \text{— 1(a)}$$

where $L = N \cdot \frac{d\phi}{di} = \frac{d\lambda}{di}$ H —— 1(b)

The above eqn as of self-inductance of the circuit. For a magnetic circuit having a linear B-H relationship or with a dominating air gap, the inductance L is a constant, independent of current and depends only on the geometry of ckt elements and permeability of the medium. In this the above eqn can also be expressed as

$$L = \lambda / i \quad \text{—— (2)}$$

The inductance can be written in terms of field quantities as, $\frac{N\phi}{i} = \frac{NBA}{i}$, $H = \frac{Ni}{l}$

$$= \frac{N \cdot BAN}{Hl}$$

$$L = \frac{N^2 BA}{Hl} = N^2 \mu \frac{A}{l} = \frac{N^2}{S} \quad \text{—— (3)}$$

Thus self-inductance is proportional to N^2

The inductance concept is easily extendable to the mutual inductance of two coils sharing a common magnetic circuit. Thus,

$$\left.\begin{aligned} M_{12} &= \frac{\lambda_{12}}{i_2} \; H \\ M_{21} &= \frac{\lambda_{21}}{i_1} \; H \end{aligned}\right\} \quad \text{——} \quad (4)$$

where. λ_{12} = flux linkages of coil 1 due to current in coil 2.

λ_{21} = flux linkages of coil 2 due to current in coil 1.

for a bilateral magnetic ckt,

$$M = M_{12} = M_{21}$$

It can also be shown that for tight coupling, ie all the flux linking both the coils,

$$M = \sqrt{L_1 L_2} \quad \text{——} \quad (5)\,a$$

In general, $M = k\sqrt{L_1 L_2}$ —— (5) b

k = coupling coefficient (which can be at most unity)

$$\lambda = Li$$

In static magnetic configuration, L is fixed independent of time so that the induced emf is given by the eqn as

$$e = \underbrace{L \cdot \frac{di}{dt}}_{\text{statically induced emf}} + \underbrace{i \cdot \frac{dL}{dt}}_{\text{dynamically induced emf}}$$

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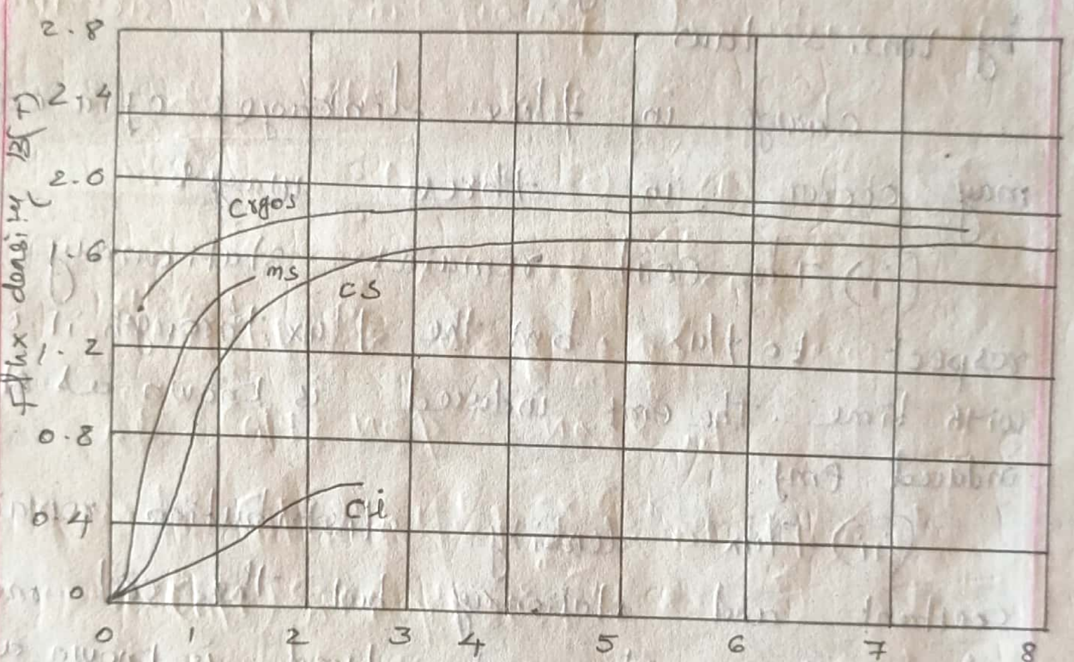
MAGNETICALLY INDUCED EMF

STATICALLY AND DYNAMICALLY INDUCED EMF.

Faraday's law of Induction, which is the integral form of the fourth Maxwell's eqn, is given as

$$\oint \vec{E} \cdot \vec{dl} = -\int \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds} \quad (1)$$



crgos - cold rolled grain oriented steel
ms - mild steel
cs - cast steel
ci - cast iron.

The integral form of eqn for a coil of N turns is

$$e = -N \frac{d\varphi}{dt} = -\frac{d\lambda}{dt} \quad (2)$$

where $\lambda = N\varphi$ = flux linkages of the coil (wb-Turns)

The positive direction of current in the coil is that direction which establishes positive flux and flux linkages. The -ve sign means that the induced emf owing to an increase in λ is in opposite direction to that of +ve current.

$$e = N \frac{d\phi}{dt} = \frac{d\lambda}{dt}$$

with the sign of the emf determined by Lenz's law.

change in flux linkages of a coil may occur in three ways.

(i) The coil remains stationary with respect to flux, but the flux through it changes with time. The emf induced is known as statically induced emf.

(ii) flux density distribution remains constant and stationary but the coil moves relative to it. The emf induced is known as dynamically induced emf.

(iii) Both changes (i) and (ii) may occur simultaneously i.e the coil moves through time varying flux. both statically and dynamically induced emfs are then present in the coil.

The dynamically induced emf in a conductor of length l placed at $90°$ to a

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magnetic field of flux density B and cutti[ng] across it at speed v is given by

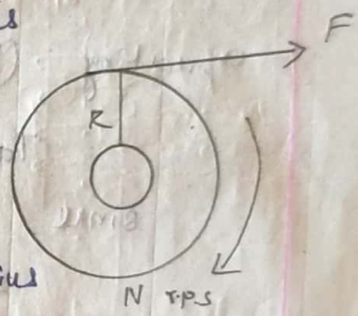
$$e = |\vec{v} \times \vec{B}| \, l = Blv \sin\theta$$

where θ is the angle between the direction of flux density and conductor velocity and l the conductor along which the flux density is assumed uniform. In electric machines θ = 90°, so that

$$\boxed{e = Blv}$$

TORQUE:

By the term torque is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts.



Consider a pulley of radius R metres acted upon by a circumferential force of F newton which causes it to rotate at N r.p.s.

Then torque, $T = F \times R$ newton-metre

workdone by this force in one revolution

$= \text{force} \times \text{distance} = F \times 2\pi R$ Joules

workdone per second $W = F \times 2\pi R \times N$ joules/second

$= (F \times R)\, 2\pi N$ joules/second

Now $2\pi N$ = angular velocity ω in radian/second

and $F \times R$ = torque T.

$\therefore$ work done / second $= T \times \omega$ joules / second

$\omega = F \times 2\pi R \times N$

power developed $= T \times \omega$ watts.

ARMATURE TORQUE OF A MOTOR.

Let T_a be the torque developed by the armature of a motor running at N r.p.s ie with an angular velocity of $\omega = 2\pi N$ rad/sec

power developed $= T_a \times \omega = T_a \times 2\pi N$ watt. ①

We also know that electrical power converted into mechanical power in the armature is

$= E_b I_a$ watts. ②

equating ① & ② we get

$$T_a \times 2\pi N = E_b I_a$$

since $E_b = \phi Z N \times (P/A)$ volts.

$$T_a \times 2\pi N = \phi Z N (P/A) \cdot I_a$$

$$T_a = \frac{1}{2\pi} \phi Z I_a (P/A) \text{ N-m}$$

$$= 0.159 \phi Z I_a (P/A) \text{ N-m}$$

$$= \frac{0.159 \phi Z I_a (P/A)}{9.81} \text{ kg-m}$$

$$= 0.0162 \phi Z I_a (P/A) \text{ kg-m}$$

From the eqn for the torque we find that $T_a \propto \phi I_a$

(a) In the case of series motor ϕ is directly proportional to I_a because field windings carry full armature current.

$$\therefore T_a \propto I_a^2$$

(b) For shunt motor ϕ is practically constant hence

$$T_a \propto I_a$$

From eqn (i) $T_a = \frac{1}{2\pi} \frac{E_b I_a}{N}$ N-m

$$= 0.159 \frac{E_b I_a}{N} \text{ N-m}$$

$$= 0.0162 \frac{E_b I_a}{N} \text{ kg-m}$$

If N is in r.p.m, then as seen from eqn (ii) above,

$$T_a = \frac{E_b I_a}{2\pi N/60} = \frac{60}{2\pi} \frac{E_b I_a}{N} = 9.55 \frac{E_b I_a}{N} \text{ N-m}$$

SHAFT TORQUE.

The torque which is available for doing useful work is known as shaft torque T_{sh}. It is so called because it is available at the shaft.

$$\text{output} = T_{sh} \times 2\pi N \text{ watt.}$$

where T_{sh} is in N-m and N in rps.

$$T_{sh} = \frac{\text{output in watts}}{2\pi N} \text{ N-m}$$

$$= \frac{\text{output in watts}}{2\pi N/60} \text{ N-m}$$

$$T_{sh} = \frac{60}{2\pi} \text{output}/N$$

The difference $T_a - T_{sh}$ is known as lost torque.

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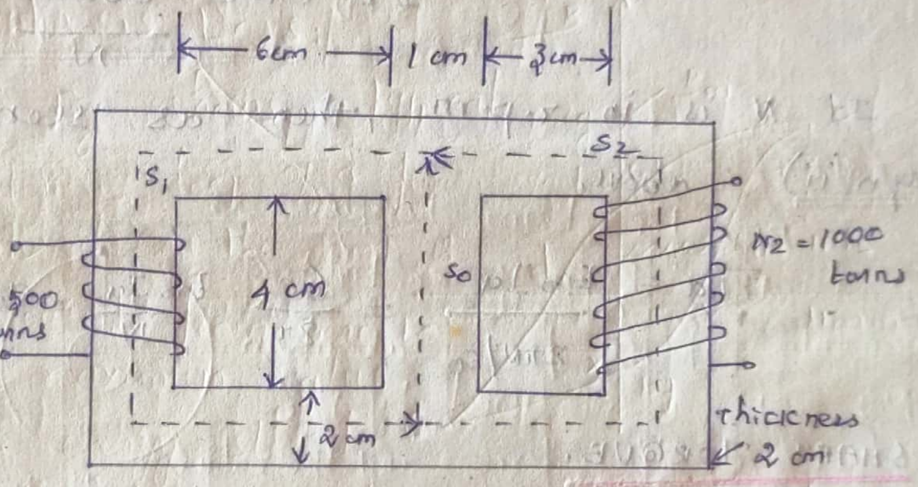
$$= 9.55 \frac{\text{output}}{N} \text{ N-m}$$

The difference $T_a - T_{sh}$ is known as lost torque.

EXAMPLE For the magnetic circuit of fig. find the self and mutual inductances b/w between the two coils. core permeability = 1600.



Soln:

$$l_1 = (6+0.5+1) \times 2 + (4+2) = 21 \text{ cm}$$

$$l_2 = (3+0.5+1) \times 2 + (4+2) = 15 \text{ cm}$$

$$l_0 = 4+2 = 6 \text{ cm}$$

$$S_1 = \frac{l_1}{\mu_0 \mu_r A} = \frac{21 \times 10^{-2}}{4\pi \times 10^{-7} \times 1600 \times 2 \times 2 \times 10^{-4}}$$

$$= 0.261 \times 10^6$$

$$S_2 = \frac{l_2}{\mu_0 \mu_r A} = \frac{15 \times 10^{-2}}{4\pi \times 10^{-7} \times 1600 \times 2 \times 2 \times 10^{-4}}$$

$$= 0.187 \times 10^6$$

$$S_0 = \frac{l_0}{\mu_0 \mu_r A} = \frac{6 \times 10^{-2}}{\ldots}$$

$= 0.149 \times 10^6$

(i) coil 1 excited with 1A

$S = S_1 + S_0 \| S_2$

$= 0.261 + 0.871 \| 0.149$

$= 0.344 \times 10^6$

$\varphi_1 = \frac{Ni}{S} = \frac{500 \times 1}{0.344 \times 10^6} = 1.453 \text{ mwb.}$

$\varphi_{21} = \varphi_2 = \cancel{1.453 \times 0.149}\ \frac{\varphi_1 \times S_0}{S_2 + S_0}$

$= \frac{1.453 \times 0.149}{0.149 + 0.187} = 0.64 \text{ mwb.}$

$L_{11} = N_1 \varphi_1 = 500 \times 1.453 \times 10^{-3}$

$= 0.7265 \text{ H}$

$M_{21} = N_2 \varphi_{21} = 1000 \times 0.649 \times 10^{-3}$

$M_{21} = 0.64 \text{ H.}$

(ii) coil 2 excited with 1A

$S = S_2 + ((S_0 S_1) / (S_0 + S_1))$

$= 0.187 + [(0.149 \times 0.281) / (0.149 + 0.281)] \times 10^6$

$S = 0.284 \times 10^6$

$\varphi_2 = \frac{N_2 \times i}{S} = \frac{(1000 \times 1)}{0.284 \times 10^6}$

$= 3.52 \text{ mwb.}$

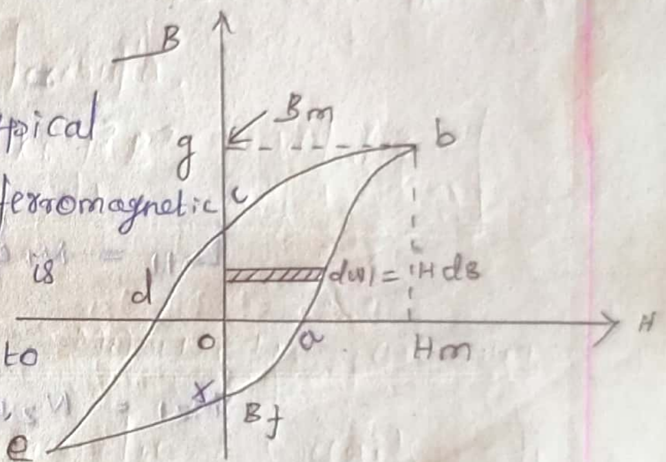
$L_{22} = N_2 \varphi_2 = 1000 \times 3.52 \times 10^{-3} = 3.52 \text{ H}$

$M_{12} = M_{21}$ (bilateral) $= 0.64 \text{ H}$

HYSTERESIS AND EDDY-CURRENT LOSSES.

When a magnetic material undergoes cyclic magnetization, two kinds of power losses occur in it, hysteresis and eddy-current losses which together are known as core loss. The core loss is important in determining transformers, machines and other ac run magnetic devices.

Hysteresis Loss:



Hysteresis loss

Fig shows a typical hysteresis loop of a ferromagnetic material. As the mmf is increased from zero to its maximum value, the energy stored in the field per unit volume of material is

$$\int_{(-B_f)}^{B_b = B_m} H \cdot dB = \text{area of abgo}$$

As H is reduced to zero, dB being negative, the energy is given out by the magnetic field and has a value

$$\int_{B_b = B_m}^{B_c} H \cdot dB = \text{area cbg}$$

The net energy recovered in the process is area of abco which is lost irretrievably in the form of heat and is called the hysteresis loss. The total hysteresis loss in one cycle is easily seen to be the area of the complete loop and let it be indicated as W_h. Then hysterisis loss in volume V of material when operated at f Hz is

$$P_h = W_h V f \text{ Watts}.$$

According from experimental studies

$$\boxed{P_h = K_h f B_m^n \quad W/m^3}$$

where, K_h = characteristic constant of the core material

B_m = maximum flux density

n = Steinmetz exponent

EDDY CURRENT LOSS

When a magnetic core carries a time-varying flux, voltage are induced in all possible paths enclosing the flux. The result is the production of circulating currents in the core. These currents are known as eddy-currents and have power loss associated with them called eddy current loss.

Higher resistivity and longer paths increase the effective resistance voltages resulting in reduction of eddy current loss. High resistivity is achieved by adding silicon to steel is used for cores conducting alternating flux. Increase the paths length of the circulating currents with consequent reduction in eddy-current loss.

The eddy current loss can be expressed by the empirical formula,

$$\boxed{P_e = K_e f^2 B^2 \quad W/m^3}$$

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ENERGY IN MAGNETIC SYSTEM.

The unique feature of electrical energy is the case with which it can be transmitted over very long distances. Moreover it is mainly used as a communication link between different forms of energies like sound, light, thermal, mechanical etc. from one place to other.

This electrical energy is not available in the natural form and cannot be stored in the form of electricity.

Conversion of electromechanical energies takes place through an electric field or magnetic field, among which the magnetic field is the most suited for electro-mechanical conversion. For effective conversion of electrical energy to mechanical energy and vice versa, either

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of the fields should be a variable one.

ENERGY CONVERSION THROUGH MAGNETIC FIELD.

Let us consider an iron core, a portion of which is movable and hinged on to one end. The non moving part of the core is wound with a coil to which an uniform electric field is applied.

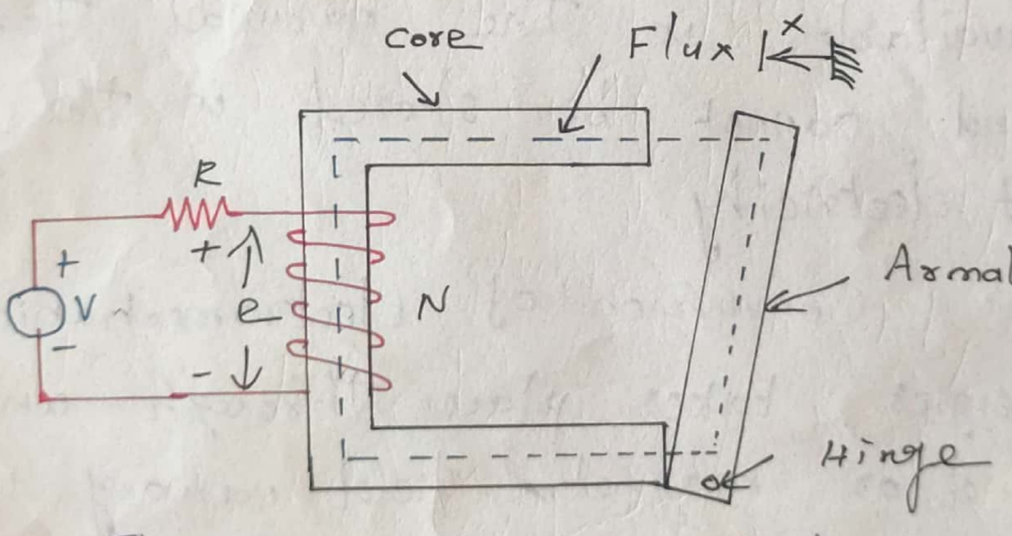

Fig shows an attracted armature relay coil. If the leakage flux is not taken into account, then it is assumed that all the flux links the coil with totally N turns.

No. of turns in the coil = N

Flux linked to the coil per turn = φ

$\therefore$ Total flux linkages $\lambda = N\varphi$ —— ①

due to the flux established in the magnetic ckt the coil experiences an induced emf in it the polarity of which is given by Lenz's law. This emf is expressed as $e = \frac{d\lambda}{dt}$ —— ②

The circuit eqn can be written for the electric ckt as $v = Ri + e$ —— ③

From eqn ② $v = Ri + \frac{d\lambda}{dt}$ —— ④

If the resistance of the coil is not considered, the total electrical power input to the coil can be written as, $P_e = \frac{dW_e}{dt} = ei$

For a small time duration dt, the energy can be given as $dW_e = ei\,dt$

From eqn (2) $dW_e = \frac{d\lambda}{dt} \cdot i \cdot dt$

$dW_e = i\,d\lambda$ —— ⑤

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As the coil is ideal no power is extracted from the coil,

$$dw_e = ei\,dt = i\,d\lambda = dw_f \quad \text{--- (6)}$$

From eqn(1) $dw_e = i\,d\lambda = i\,N d\varphi = dw_f$ --- (7)

But the total magnetomotive force can be considered as the product of number of turns and current.

ie $\mho = Ni$ --- (8)

sub. eqn 8 & 7

$$dw_e = dw_f = \mho\,d\varphi \quad \text{--- (9)}$$

For a finite change in flux linkages, the stored energy becomes

$$\boxed{\Delta w_f = \int_{\varphi_1}^{\varphi_2} \mho \cdot d\varphi} \quad \text{--- (10)}$$

This energy is the magnetic energy stored in the coil got by conversion of electric energy.

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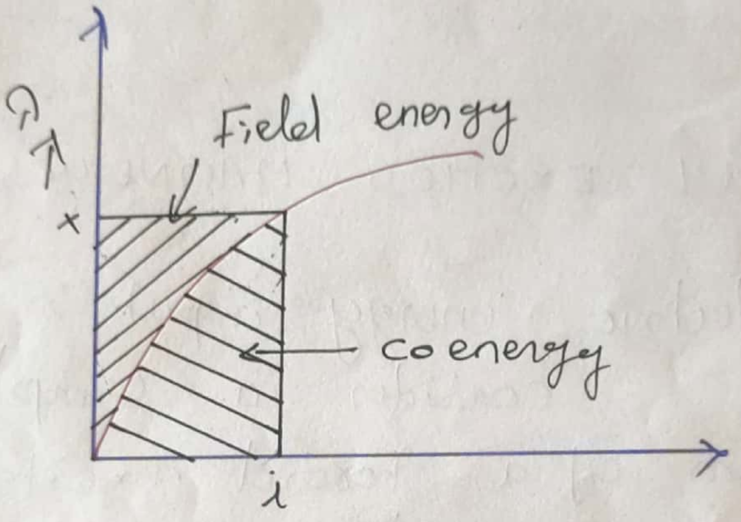
CO- ENERGY

The electrical energy given to the coil is stored in the form of magnetic energy represented by W_f.



If a graph is plotted between current and flux linkages, the area covered by $i\lambda$ gives the equivalent of total input energy to the coil. Now the total amount of this energy is stored as magnetic energy. Some quantum of energy is also utilized for some other energy conversions which constitute the co-energy. Coenergy is the complementary of field energy.

$$\therefore W_f(\lambda, x) = \lambda i - W_f(\lambda, x) \quad \text{--- (1)}$$

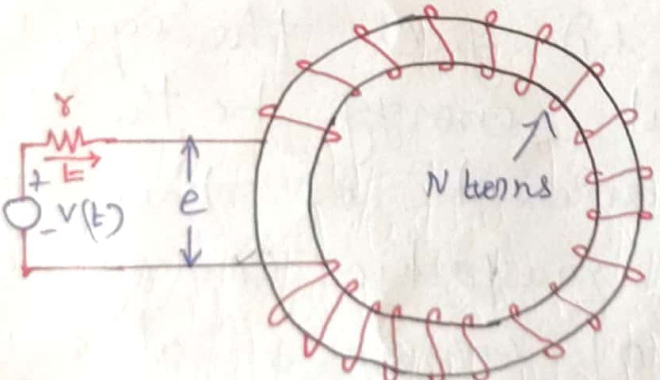
If $\lambda = \lambda(i, x)$ then

$$W_f'(i, x) = \lambda i - W_f(i, x) \quad \text{--- (2)}$$

SINGLY EXCITED MAGNETIC SYSTEMS.

(a) Electric energy input.

consider a simple magnetic system of a toroid excited by a single coil shown in fig. The instantaneous voltage equation for the



electric circuit is written by applying kirchoff's voltage law

$$v(t) = ir + e$$

where e is the reaction emf taken

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as a voltage drop in the direction of current ie, $\therefore e = \frac{d\psi}{dt}$

and $v_t = ir + \frac{d\psi}{dt}$

Here ψ is the instantaneous flux linkages with the ckt.

Multiplying both sides of eqns by idt, we get

$$v_t \cdot i \cdot dt = ri^2 \cdot dt + i\,d\psi$$

$$(v_t - ir)\,i\,dt = i\,d\psi$$

$$e\,i\,dt = i\,d\psi$$

$$dW_{elec} = e\,i\,dt = i\,d\psi$$

The flux linkages ψ are equal to $N\phi$ wb-turns.

$$\therefore dW_{elec} = i \cdot d\psi = iN\,d\phi = \underset{mmf}{F} \cdot d\phi$$

(b) Magnetic field Energy stored.

consider a simple magnetic relay Initially the armature is in the open position. when switch S is closed, current i is established in the

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N - turn coil. The flux set up depends on m.m.f Ni and the reluctance of the magnetic path. If the armature is not allowed to move, the mechanical work done, dW_{mech} is zero.

$$\therefore dW_{elec} = 0 + dW_{fld}$$

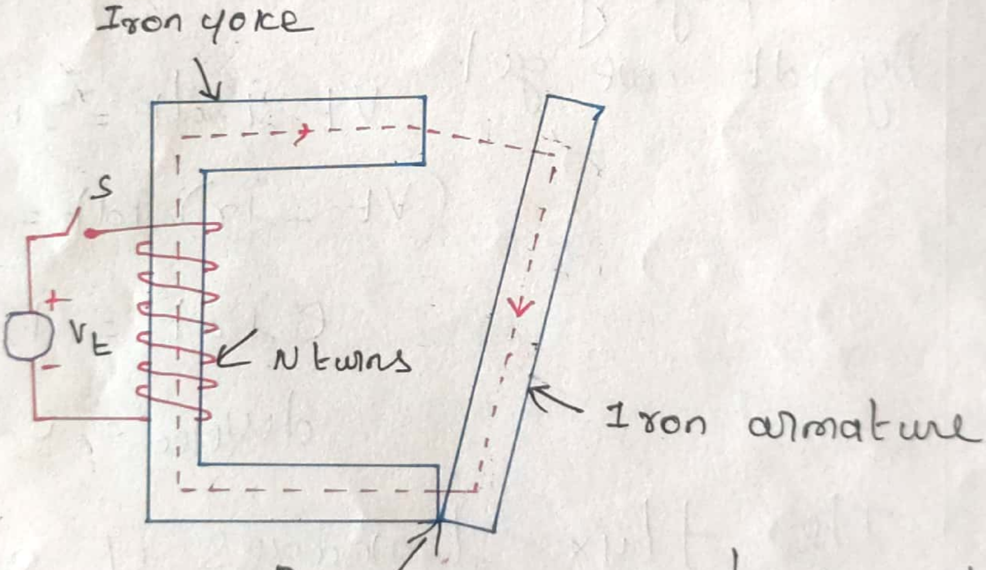

Fig. simple Magnetic relay.

This shows that when the movable part of any physical system is kept fixed, the entire electrical energy input is stored in the magnetic field.

$$dW_{field} = dW_{elec}.$$

From the above eqn $dW_{fld} = dW_{elec} = i\,d\psi$
$= F \cdot d\phi$

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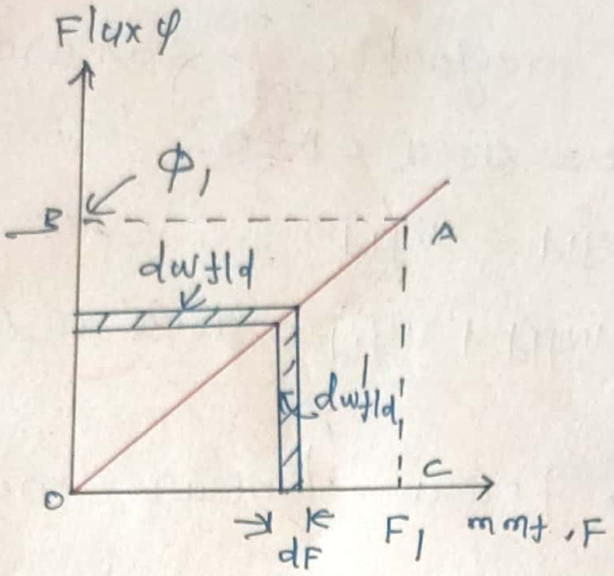

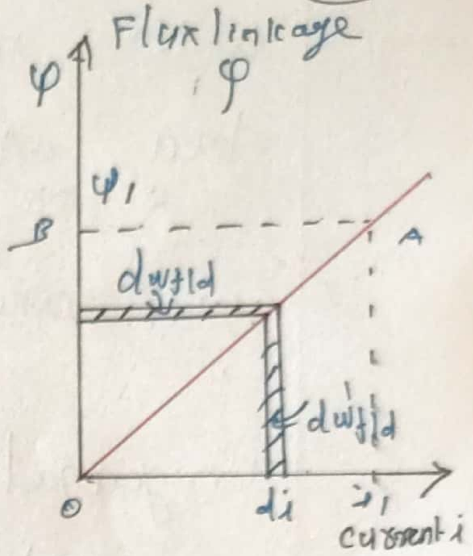

If the initial flux is zero, then the magnetic field energy stored W_{fld} in establishing a flux Φ_1 or flux linkage φ_1 is given by,

$$W_{fld} = \int_0^{\varphi_1} i \cdot d\varphi = \int_0^{\Phi_1} F \cdot d\phi$$

From fig. $W_{fld} = \int_0^{\varphi_1} dw_{fld} = \int_0^{\Phi_1} F \cdot d\phi$

$= \text{area } OABO.$

$$W_{fld} = \int_0^{\varphi_1} dw_{fld} = \int_0^{\varphi_1} i \cdot d\varphi$$

$= \text{area } OABO$

$$\text{Area } OACO = \int dw'_{fld} = \int_0^{F_1} \phi \, d \cdot F = \int_0^{i_1} \varphi \, di$$

This area OACO is called the co energy W'_{fld}.

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with no magnetic saturation.

Area $OABO$ = area $OACO$

or $W_{fld} = W'_{fld}$

and $W_{fld} + W'_{fld}$ = area $OCABO = \phi_1 F_1$

$= \psi_1 i_1$

In general, for a linear magnetic circuit, $W_{fld} = W'_{fld} = \frac{1}{2}\psi i = \frac{1}{2} F\phi$

Now mmf = flux × reluctance

$$F = (\phi) R_l = \frac{\phi}{\text{permeance}, \Delta}$$

$$W_{fld} = W'_{fld} = \frac{1}{2}\phi^2 R_l = \frac{1}{2}\frac{\phi^2}{\Delta}$$

Also $W_{fld} = W'_{fld} = \frac{1}{2} F^2 \Delta = \frac{1}{2}\frac{F^2}{R_l}$

The self-inductance L is defined as the magnetic flux linkages per ampere i.e., $L = \frac{\psi}{i}$

$$W_{fld} = W'_{fld} = \frac{1}{2} L i^2 = \frac{1}{2}\frac{\psi^2}{L}$$

Summarising the results obtained for a linear magnetic circuit, the stored magnetic energy W_{fld} and co-energy W'_{fld} can be written

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as follows :

$$W_{fld} = W'_{fld} = \tfrac{1}{2} F \Phi = \tfrac{1}{2} \psi i = \tfrac{1}{2} \Phi^2 R_L$$

$$= \tfrac{1}{2} \frac{\Phi^2}{\Lambda}$$

$$= \tfrac{1}{2} \frac{F^2}{R_l} = \tfrac{1}{2} F^2 \Lambda = \tfrac{1}{2} L i^2$$

$$\boxed{W_{fld} = \tfrac{1}{2} \frac{\psi^2}{L}} \text{ joules.}$$

MULTIPLY - EXCITED SYSTEMS.

When more than one excitation coil is used there comes the interaction between the two excitation coils. Hence apart from self inductance, Mutual inductance should also be taken in to consideration.

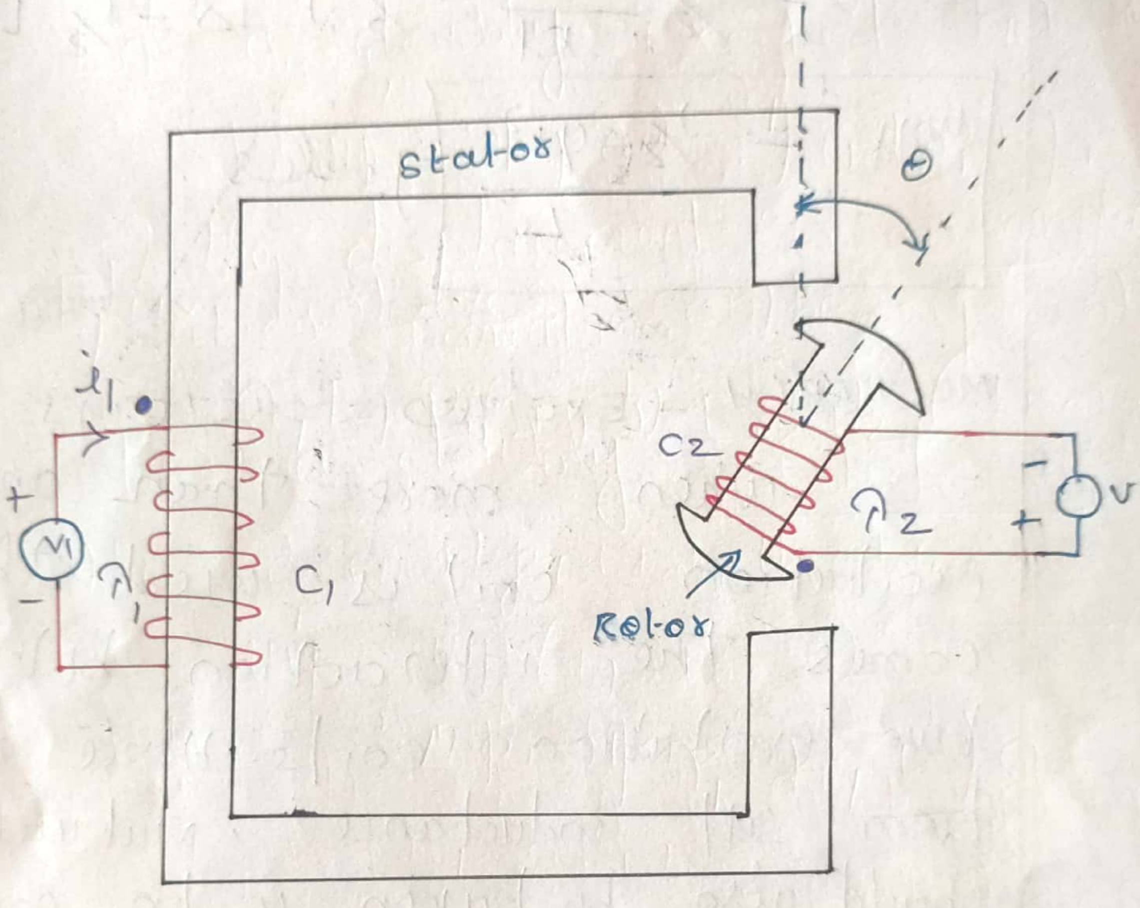
For analysis, let us consider a magnetic system with two energizing coils one in the stator and other in the rotor.

Let $C_1, C_2 \rightarrow$ coils wound on the stator and rotor respect -ively

$V_1, V_2 \rightarrow$ Excitation voltages.

$i_1, i_2 \rightarrow$ currents through the coils C_1, C_2 respectively.

$\lambda_1, \lambda_2 \rightarrow$ flux linkages



Flux linkage as independent variables

considering the flux linkages λ_1, λ_2 and angle θ as the independent variables from eqn,

$$F_f = -\frac{\partial W_f}{\partial x}$$

$$F_f \cdot \partial x = -\partial W_f$$

If the variation in distances is

7

angular position then

$$T_f = F_f \partial\theta = -\partial W_f(\lambda_1, \lambda_2, \theta) \quad \text{--- (1)}$$

For a doubly excited act shown.

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} i_1 \, d\lambda_1 + \int_0^{\lambda_2} i_2 \, d\lambda_2 \quad \text{--- (2)}$$

Flux linkages can be represented as

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \quad \text{--- (3)}$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2 \quad \text{--- (4)}$$

$$i_1 = B_{11} \lambda_1 + B_{12} \lambda_2 \quad \text{--- (5)}$$

$$i_2 = B_{21} \lambda_1 + B_{22} \lambda_2 \quad \text{--- (6)}$$

sub 5 and 6 in (2)

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} i_1 \, d\lambda_1 + \int_0^{\lambda_2} i_2 \, d\lambda_2$$

$$\boxed{W_f(\lambda_1, \lambda_2, \theta) = B_{11} \frac{\lambda_1^2}{2} + B_{22} \frac{\lambda_2^2}{2} + B_{12} \lambda_1 \lambda_2}$$

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currents as independent variable.

Considering the coil magnetization currents i_1, i_2 and angular displacement θ as independent variables

$$\partial W_f' = F_f \, \partial x$$

If the displacement is an angular position variation then

$$F_f \, \partial \theta = T_f = \partial W_f'(i_1, i_2, \theta)$$

$$\text{———— } (1)$$

For a doubly excited system the coenergy can be given as,

$$W_f' = W_f'(i_1, i_2, \theta)$$

$$W_f' = \int_0^{i_1} \lambda_1 \cdot di_1 + \int_0^{i_2} \lambda_2 \cdot di_2$$

$$\text{———— } (2)$$

$$\boxed{W_f'(i_1, i_2, \theta) = \frac{L_{11} i_1^2}{2} + \frac{L_{22} i_2^2}{2} + L_{12} i_1 i_2}$$

$$\text{—— } (3).$$

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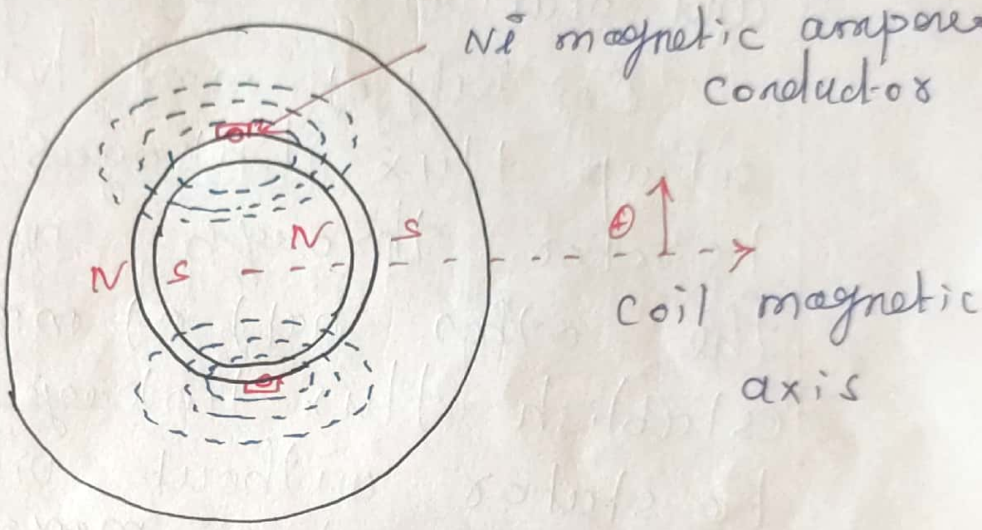
MMF IN DISTRIBUTED WINDING.

MMF in a single coil winding:

Let the coils be full-pitched coils with each coil having N

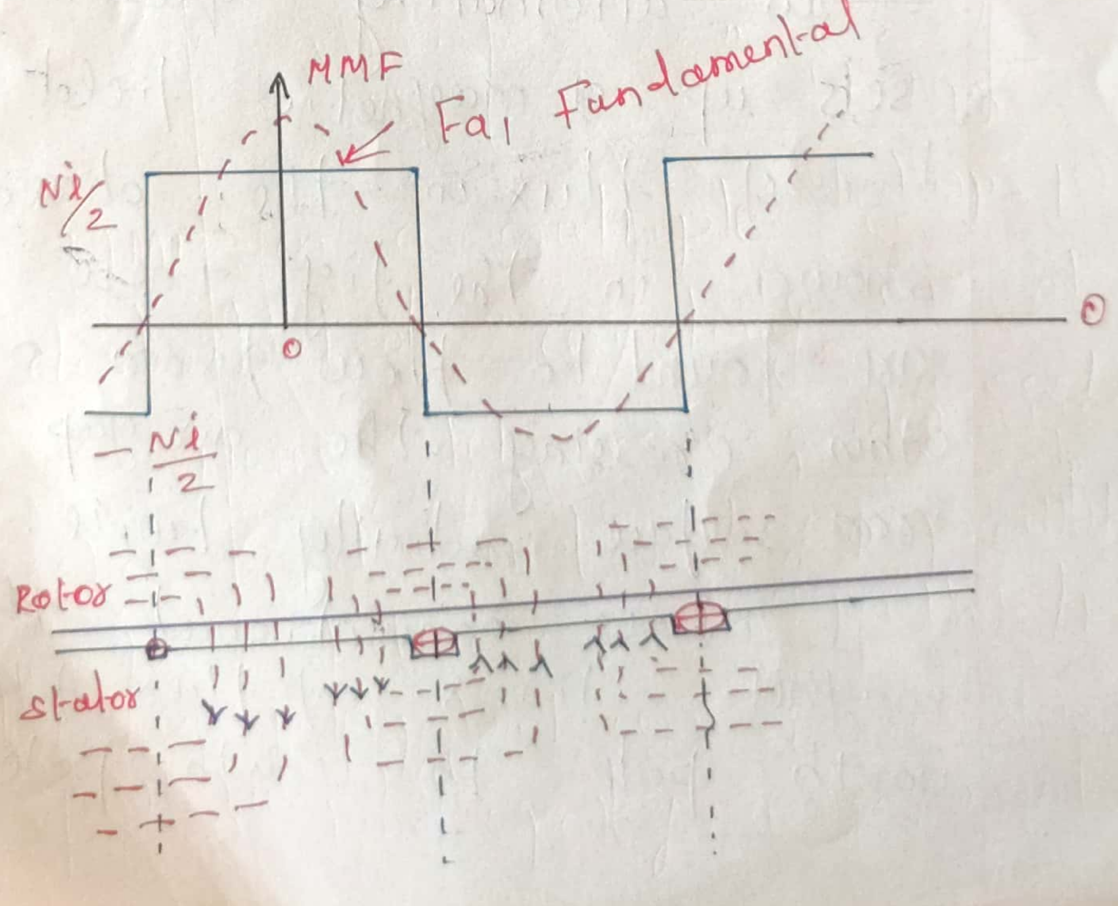


numbers of turns. As the current is an alternating quantity, it sets up a magnetic field with lines of flux in the direction shown in the fig. These lines are found to flow from one end to other crossing the air gap and rotor iron surface radially twice.

It is conventional that always magnetic lines of flux flow from north pole to south pole.

For the machine to be maintained at synchronous speed, the rotor iron surface should have opposite poles induced so as to get attracted and rotation in synchronism.

One half of mmf $Ni/2$ is used to set up flux linkages from stator to rotor through air gaps and the other half of mmf is used to establish flux linkages from rotor to stator without violating the properties of a magnetic field.



Total change in mmf for the flux to link stator end to end in any slot is given by,

$$\Delta mmf = Ni/2 - (-Ni/2)$$

$$\boxed{\Delta mmf = Ni}$$

the fundamental becomes

$$mmf_{fa} = \frac{4}{\pi} \frac{Ni}{2} \cos\theta$$

$$AT = F_p \cos\theta$$

where $\boxed{F_p = \frac{4}{\pi}\left(\frac{Ni}{2}\right)}$

MMF in a multiple coil distributed winding.

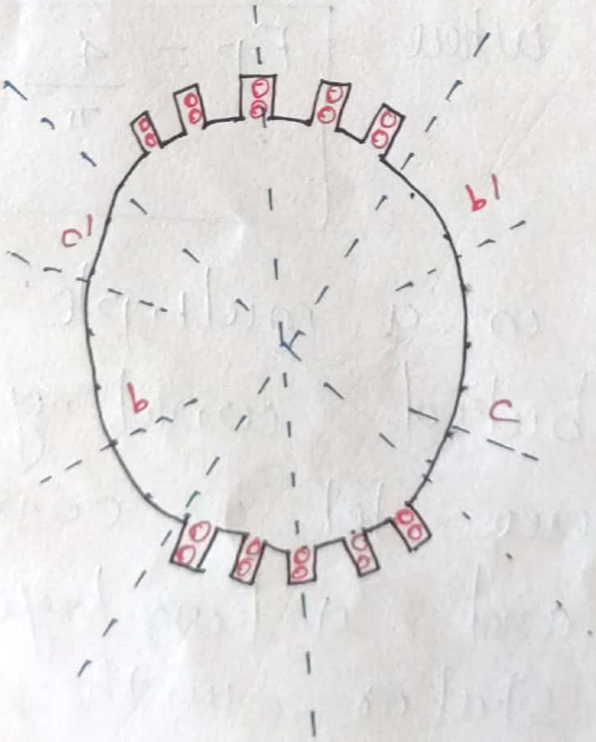
Here let us consider a wound rotor and a two layer winding in the stator with q slots / pole / ph.

If the machine is designed for a two pole arrangement.

then half the ampere conductors of the middle slot of a phase group contributes to establish say north pole and vice versa.

∴ The fundamentals of these mmf waves are out of phase by

$$\boxed{\gamma = \frac{\pi p}{S}} \text{ rads.}$$



If N_{ph} – number of turns per phase

i_c – current in a coil

A — Number of parallel path.

Fig. two pole structure with two-layer winding.

then AT / parallel path = $N_{ph} \cdot i_c$

AT / phase = $N_{ph} \cdot i_c \cdot A$

$i_a = i_c \cdot A$

AT / ph / pole = $\frac{N_{ph}}{P} i_a$

peak value of fundamental

$$F_P = \frac{4}{\pi} \frac{N_{ph}}{P} i_a k_b$$

$$F_a = F_p \cos\theta$$

$$\boxed{F_a = \frac{4}{\pi} \frac{N_{ph}}{P} i_a k_b \cos\theta}$$

if chorded windings are used then the peak value of the mmf gets reduced by the pitch factor k_p.

$$\boxed{F_a = \frac{4}{\pi} \frac{N_{ph}}{P} i_a k_w \cos\theta}$$

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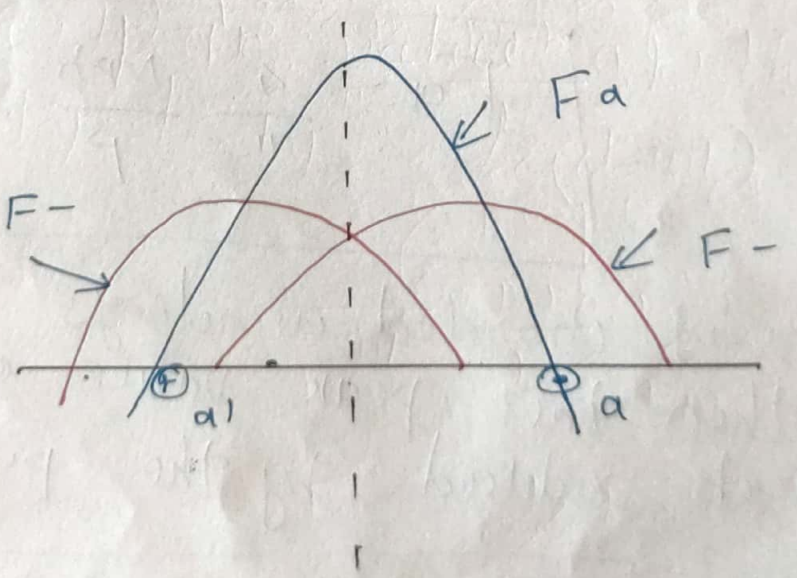
MAGNETIC FIELD IN ROTATING MACHINES.

Let us consider the 3ϕ balanced supply allows the following balanced currents to flow through the windings as

$$i_a = I_m \cos\omega t$$

$$i_b = I_m \cos(\omega t - 120^\circ)$$

$$i_c = I_m \cos(\omega t - 240^\circ)$$



These develops mmfs that can be expressed as follows.

$$F_a = F_m \cos\omega t \cos\theta$$

$$F_b = F_m \cos(\omega t - 120^\circ) \cos(\theta - 120^\circ)$$

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$F_c = F_m \cos(\omega t - 240°) \cos(\theta - 240°)$

The resultant mmf is given by

$$F = F_a + F_b + F_c$$

$$F(\theta, t) = F_m \cos \omega t \cos\theta + F_m \cos(\omega t - 120)\cos(\theta - 120) + F_m \cos(\omega t - 240)\cos(\theta - 240)$$

$$\boxed{F(\theta, t) = \frac{3}{2} F_m \cos(\theta - \omega t)}$$

Here the peak value of the mmf developed $F_p = \frac{3}{2} F_m$

$$F_p = \frac{3}{2}\left[\frac{4\sqrt{2}}{\pi} \frac{N_{ph}}{P} k_w I_{rms}\right]$$

$$F(\theta, t) = \frac{3 \cdot 2\sqrt{2}}{\pi} \frac{N_{ph}}{P} k_w I_{rms} \cos(\theta - \omega t)$$

$$F(\theta, 0) = \frac{3 \cdot 2\sqrt{2}}{\pi} \frac{N_{ph}}{P} k_w I_{rms} \cos\theta$$

ROTATING MMF WAVES.

Let us consider a vector $\vec{F_a}$ representing the mmf along the phase 'a' with an amplitude equal to peak value F_m along the positive axis of phase 'a'.

If mmf varies both in space and time, then $\vec{F_a} = F_m \cos\omega t + \cos\theta$

$$= \frac{1}{2} F_m \left[\cos\omega t \cos\theta + \cos\omega t \cos\theta\right\}$$

$$\vec{F_a} = \frac{1}{2} F_m \left\{\cos\theta + \cos\omega t - \sin\theta \sin\omega t\right\} +$$

$$\frac{1}{2} F_m \left[\cos\theta + \cos\omega t + \sin\theta \sin\omega t\right\}$$

$$\vec{F_a}' = \frac{1}{2} F_m \cos(\theta + \omega t)$$

$$\vec{F_a}'' = \frac{1}{2} F_m \cos(\theta - \omega t)$$

$$\vec{F_a} = \vec{F_a}' + \vec{F_a}''$$

F_a' rotates in +ve direction F_a'' rotates in negative direction.

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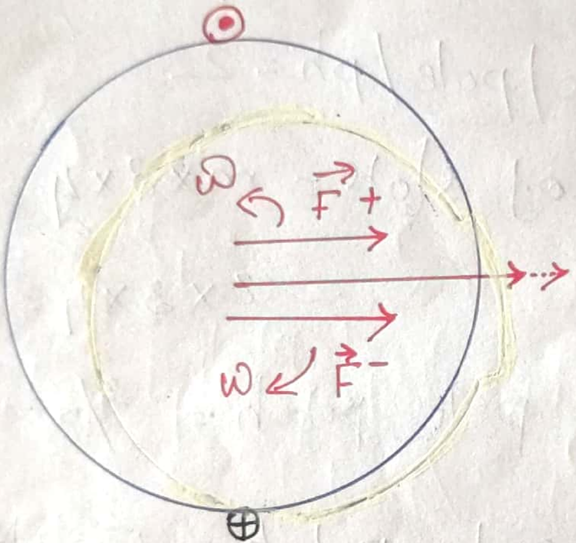
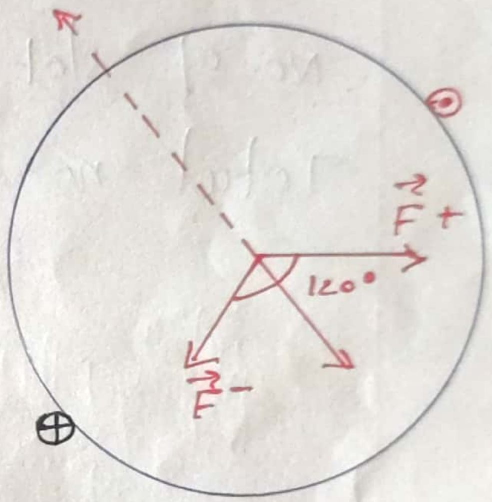
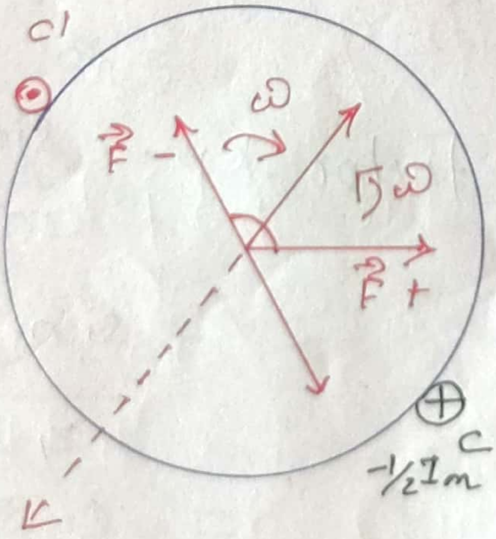
This shows that pulsating mmf can be resolved into two components Each rotating in a direction opposite to each other.



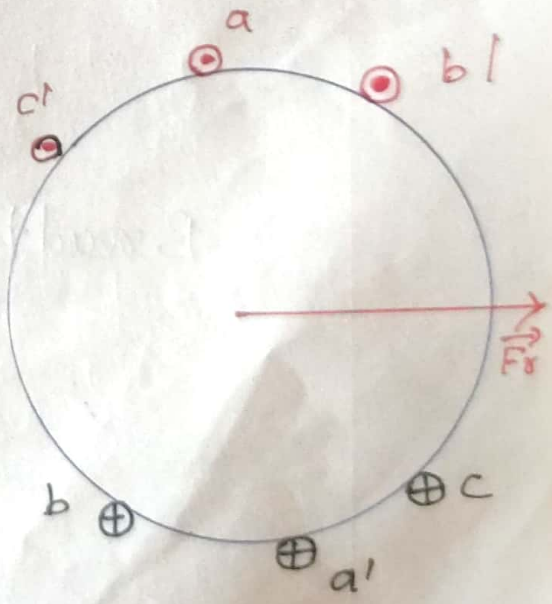
Field of phase a



Field of phase b



axis of phase c



Resultant field

Example Determine the breadth and pitch factors for a 4-pole 3ϕ winding with 2 slots/pole/phase if the coil span is 5 slot pitches.

Soln:

no of poles = 4

No. of slots/pole/ph = 2

Total no. of slots = $m \times 3 \times 4$

$= 2 \times 3 \times 4 = 24$

pole pitch $= S/P = \frac{24}{4} = 6$

coil pitch = 5 slot pitches

slot angle $\gamma = \pi P/S = \frac{180 \times 4}{24}$

$= 30^\circ$

Breadth factor $K_b = \frac{\sin m\gamma/2}{m \sin \gamma/2}$

$= \frac{\sin 4 \times 30/2}{4 \sin 30/2}$

$= 0.8365$

pitch factor $K_p = \cos(\theta_{sp}/2)$

$\theta_{sp} = (6-5)30^\circ$

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$K_p = \cos 15° = 0.966$

$$\boxed{K_b = 0.8365,\ K_p = 0.966}$$

Example 2: A 3φ, 400 KVA, 50 Hz star connected synchronous generator running at 300 rpm is designed to develop 3300V between terminals. The armature consists of 180 slots, each slot having one coil side with eight conductors. Determine the peak value of the fundamental mmf in AT/pole when the machine is delivering full load current.

Soln:

$$\text{speed} = \frac{120f}{P} = 300\,\text{rpm}$$

$$P = \frac{120 \times 50}{300} = 20.$$

Line voltage $= 3300V$, $P_o = 400\,kVA$

$$\therefore \sqrt{3}\, V_L I_L = P_o = 400 \times 10^3$$

$$I_L = \frac{400 \times 10^3}{\sqrt{3} \times 3300} = 70\,\text{Amps.}$$

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$I_L = I_{ph} = 70$ Amps.

No. of slots $S = 180$

No. of conductors / slot $= 8$

$\therefore$ No of turns $= \frac{S \times 8}{2} = \frac{180 \times 8}{2} = 720$

$\therefore$ No. of turns / ph $= N_{ph} = \frac{720}{3} = 240$

Slot angle $\gamma = \frac{\pi P}{S} = \frac{180 \times 20}{180}$

$= 20°$

slot / pole / ph $m = \frac{S}{3P} = \frac{180}{3 \times 20}$

$m = 3$

The breadth factor $= \frac{\sin(m\gamma/2)}{m \sin(\gamma/2)}$

$= \frac{\sin (3 \times 20)/2}{3 \sin(20/2)}$

$K_b = 0.9597$

MMF peak value per pole is given,

$AT = 3/2 \; F_m = 3/2 \left\{ \frac{4\sqrt{2}}{\pi} K_b \frac{N_{ph}}{P} I_{ph} \right\}$

$= 3/2 \left\{ \frac{4\sqrt{2}}{\pi} \times 0.9597 \times \frac{240}{20} \times 70 \right\}$

$\boxed{AT_{peak} = 2177.365 \text{ AT/pole}}$

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⇒ A ring composed of three sections. The cross sectional area is $0.001 \, m^2$ for each section. The mean arc length are $l_a = 0.3 \, m$, $l_b = 0.2 \, m$, $l_c = 0.1 \, m$. An air gap length of $0.1 \, mm$ is cut in the ring. Relative permeability for sections a, b and c are 5000, 1000 and 10,000 respectively. Flux in the air gap is 7.5×10^{-4} wb. Find,

(i) mmf (ii) exciting current if the coil has 100 turns (iii) reluctance of the sections

(April/May 2019)

soln:-

(i) Total mmf $= H_g l_g + H_A l_A + H_B l_B + H_C l_C$

$$= \frac{B}{\mu_0} l_g + \frac{B_A}{\mu_0 \mu_{rA}} l_A + \frac{B_B}{\mu_0 \mu_{rB}} l_B + \frac{B_C}{\mu_0 \mu_{rC}} l_C$$

$B = \mu H$

$\varphi = BA$

$$= \frac{\varphi}{\mu_0 A_g} l_g + \frac{\varphi}{\mu_0 \mu_{rA} A_A} \cdot l_A + \frac{\varphi}{\mu_0 \mu_{rB} A_B} \cdot l_B + \frac{\varphi}{\mu_0 \mu_{rC} A_C} \cdot l_C$$

$$= \frac{\varphi}{\mu_0 A} \left(l_g + \frac{l_A}{\mu_{rA}} + \frac{l_B}{\mu_{rB}} + \frac{l_C}{\mu_{rC}} \right)$$

$$= \frac{7.5 \times 10^{-4}}{4\pi \times 10^{-7} \times 0.001} \left(0.0001 + \frac{0.3}{5000} + \frac{0.2}{1000} + \frac{\text{[illegible]}}{10000} \right)$$

$$= 220.83 \text{ AT}$$

(ii) Exciting current

Total mmf = NI

$$I = \frac{mmf}{N} = \frac{220.83}{100} = 2.2 \text{ A}$$

(iii) Reluctance of each section:

$$S_A = \frac{l_A}{\mu_0 \mu_r A_A} = \frac{0.3}{4\pi \times 10^{-7} \times 5000 \times 0.001} = 47.75 \times 10^3 \text{ AT/wb}$$

$$S_B = \frac{l_B}{\mu_0 \mu_r A_B} = \frac{0.2}{4\pi \times 10^{-7} \times 1000 \times 0.001} = 159.15 \times 10^3 \text{ AT/wb}$$

$$S_C = \frac{l_C}{\mu_0 \mu_r A_C} = \frac{0.1}{4\pi \times 10^{-7} \times 10{,}000 \times 0.001} = 7.96 \times 10^3 \text{ AT/wb}$$

$$S_g = \frac{l_g}{\mu_0 \mu_r A_g} = \frac{0.1 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 0.001} = 79.58 \times 10^3 \text{ AT/Wb.}$$

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A square - wave voltage of amplitude $E = 100$ V and frequency 60 Hz is applied on a coil wound on a closed iron core. The coil has 500 turns, and the cross - sectional area of the core is $0.001\,m^2$, Assume that the coil has no resistance.

(i) Find the maximum value of the flux and sketch the waveforms of voltage and flux as a function of time.

(ii) Find the maximum value of E if the maximum flux density is not to exceed 1.2 Tesla.

(Nov / Dec 2018)

soln :-

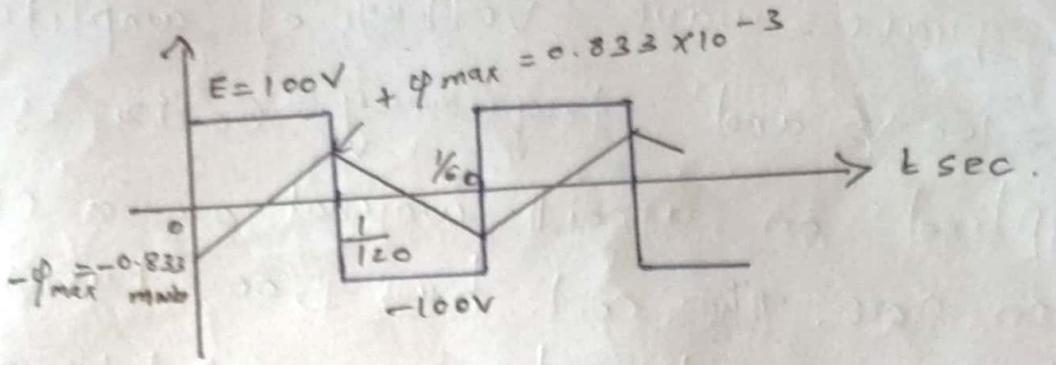
(a) $$e = N \cdot \frac{d\varphi}{dt}$$

$$N \cdot d\varphi = e \cdot dt$$

$$N \cdot \Delta\varphi = E \cdot \Delta t$$

Flux linkage change = volt - time product

$$500(2\varphi_{max}) = E \times \frac{1}{120}$$



$$\varphi_{max} = \frac{100}{1000 \times 120} \text{ wb}$$

$$= 0.833 \times 10^{-3} \text{ wb.}$$

(b) $B_{max} = 1.2\,T$

$$\varphi_{max} = B_{max} \times A$$

$$= 1.2 \times 0.001 = 1.2 \times 10^{-3} \text{ wb.}$$

$$N\,(2\varphi_{max}) = E \times \frac{1}{120}$$

$$E = 120 \times 500 \times 2 \times 1.2 \times 10^{-3}$$

$$\boxed{E = 144V}$$

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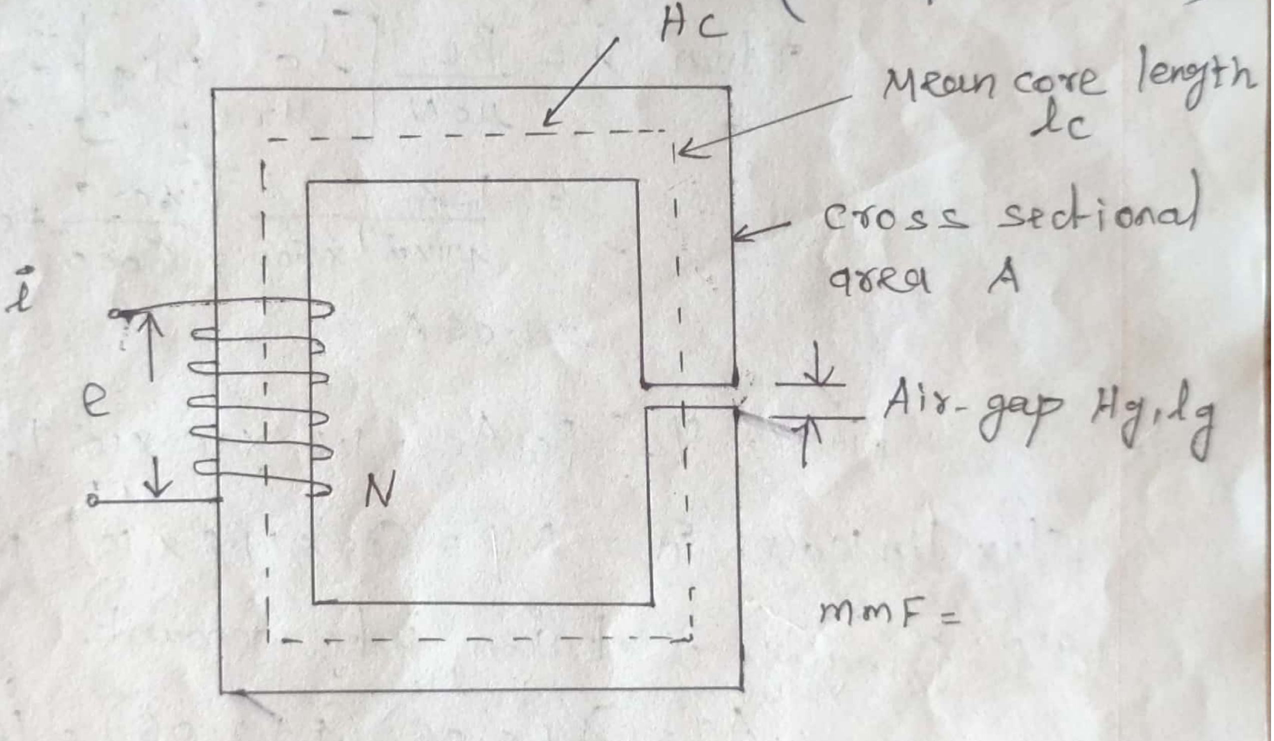
Eg: The magnetic circuit of fig has dimension $A_c = 4 \times 4\ cm^2$, $l_g = 0.06\ cm$, $l_c = 40\ cm$ $N = 600$ turns, Assume the value of $\mu_r = 6000$ for iron. Find the exciting current for $B_c = 1.2\ T$ and the corresponding flux and flux linkages.

(NOV/DEC 2018)



Solution:-

Given data: Area of the core $= 4 \times 4\ cm^2$

$l_g = 0.06\ cm$, $l_c = 40\ cm$, $N = 600$ turns

$\mu_r = 6000$, $B_c = 1.2\ T$.

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From fig, the ampere turns for the circuit are given by,

$$Ni = \frac{B_c}{\mu_o \mu_r} l_c + \frac{B_g}{\mu_o} l_g \quad \text{— (1)}$$

Neglecting fringing $A_c = A_g$

$$\therefore B_c = B_g$$

Then $i = \frac{B_c}{\mu_o N}\left[\frac{l_c}{\mu_r} + l_g\right]$ — (2)

$$= \frac{1.2}{4\pi \times 10^{-7} \times 600}\left[\frac{40}{6000} + 0.06\right] \times 10^{-2}$$

$$= 1.06\ A$$

Flux $\varphi = B_c A_c = 1.2 \times 16 \times 10^{-4} = 19.2 \times 10^{-4}\ wb$

Flux linkages $\lambda = N\varphi = 600 \times 19.2 \times 10^{-4} = 1.152$ wb turns

If fringing is taken into account,

$$A_g = (4 + 0.06)(4 + 0.06) = 16.484\ cm^2$$

Effective $A_g > A_c$ reduces the air gap reluctance

Now $B_g = \frac{19.2 \times 10^{-4}}{16.484 \times 10^{-4}} = 1.165\ T.$

From eqn (i) $i = \frac{1}{\mu_o N}\left[\frac{B_c l_c}{\mu_r} + B_g l_g\right]$

$$= \frac{1}{4\pi \times 10^{-7} \times 600}\left[\frac{1.2 \times 40 \times 10^{-2}}{6000} + 1.165 \times 0.06 \times 10^{-2}\right]$$

$$= 1.0332\ A.$$

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Ex: An iron rod of 1 cm radius is bent to a ring of mean diameter 30 cm and wound with 200 turns of wire. Assume the relative permeability of iron as 800. An air gap of 0.1 cm is cut across the bent ring. calculate the current required to produce a useful of 20,000 lines if (1) Leakage is neglected (2) Leakage factor is 1.1.

an:- $l_g = 0.1$ cm

$d/2 = \frac{30}{2} = 15$ cm

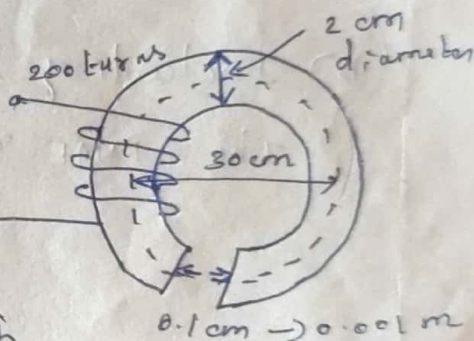
$2R_m = 30$ cm

$N = 250$ turns

$\mu_r = 800$

$\varphi = 20,000$ lines

$= 20000 \times 10^{-8}$ wb.



$[r = 1 (cm)$
$r = 1 \times 10^{-2}$ m
$a = \pi \times 1 \times 10^{-4}]$

(a) Neglecting leakage

Total Reluctance = Reluctance of air gap + Reluctance of iron path.

Reluctance of air gap $= \frac{l_g}{\mu_0 \mu_r a} = \frac{0.001}{4\pi \times 10^{-7} \times 1 \times \pi \times 1 \times 10^{-4}}$

$= 2533029.59$

l_i, Length of iron path = Total length − l_g

$l_i = (\pi d - l_g) = (\pi \times 0.3 - 0.001)$

Reluctance of iron path $= \frac{\pi \times 0.3 - 0.001}{4\pi \times 10^{-7} \times 800 \times \pi \times 1 \times 10^{-4}}$

$= 2980988.896 \quad A/wb$

Total reluctance $= 5514018.49 \quad A/wb.$

MMF = Flux × Reluctance

$= 201000 \times 10^{-8} \times 5514018.49$

$= 1102.8$ Amps turns

Current required $= \dfrac{1102.8}{\text{No. of Amp. turns}}$

$= \dfrac{1120.8}{250}$

Current required $= 4.41$ Amps

(b) Including leakage.

The MMF required on the air gaps portion

$= \varphi \times$ Reluctance of air gap

$= 201000 \times 10^{-8} \times 2533039.59$

MMF for air gap $= 506.606$ Amp.

For the iron path the flux has iron be more

The total flux in iron path = Leakage factor × useful flux

$= 1.1 \times 20{,}000 \times 10^{-8}$ wb $\times 2980988.896$

[Useful flux = $\varphi \times$ reluct of iron path.]

MM for iron path $= 655.82$ Amp

Total MMF $= 655.82 + 506.606$

$= 1162.426$ A

Current required $= \dfrac{1162.426}{250}$

Current required $= 4.649$ Amps.

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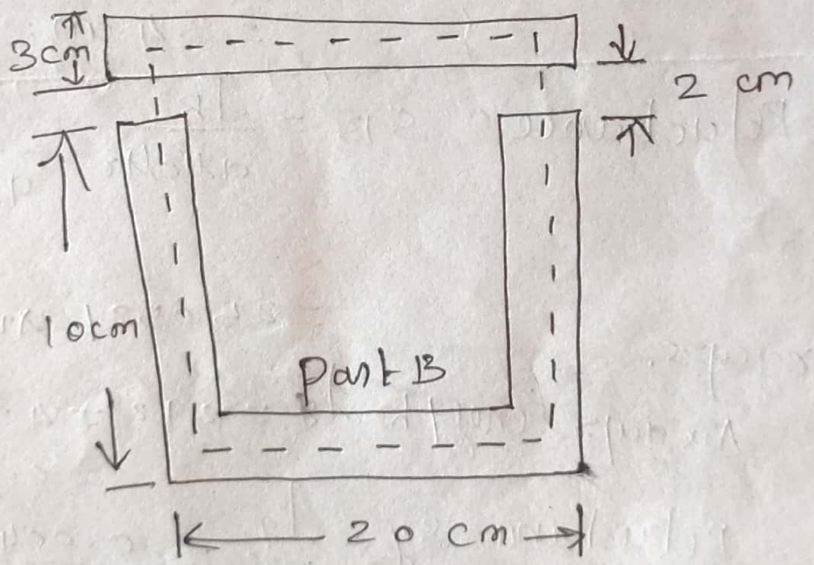
Ex: The magnetic circuit shown in fig. is built up of iron of square cross-section 3 cm wide. Each air gap is 2 mm wide. Each coil is wound with 1000 turns and exciting current is 1 A. The relative permeability of part A and part B may be taken as 1000 to 1200 respectively. Find (i) reluctance of part A (ii) Reluctance of part B (iii) reluctance of two air gaps (iv) total reluctance (v) total mmf.



Solution:-
The dotted line shows the mean path of flux.

i) Part A: Mean length $l_A = 20 - 1.5 - 1.5 + 1.5 + 1.5$

$= 20 \text{ cm}$

$= 0.2 \text{ m}$

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Area of cross section $a = 3 \times 3 = 9\, cm^2$

$= 9 \times 10^{-4}\, m^2$

Reluctance $S_A = \frac{l_A}{a \mu_0 \mu_r}$

$= \frac{0.2}{9 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1000}$

$= 176838$ AT/wb.

(ii) Part B, Mean length $l_B = 20 - 1.5 - 1.5 + 2(10 - 1.5)$

$= 34\, cm = 0.34\, m$

Reluctance $S_B = \frac{l_B}{a \mu_0 \mu_r} = \frac{0.34}{9 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1200}$

$= 250521$ AT/wb.

(iii) Airgaps.

Airgap length $l_g = 2 + 2 = 4\, mm$

$= 0.004\, m$

Reluctance $S_g = \frac{l_g}{a \mu_0} = \frac{0.004}{9 \times 10^{-4} \times 4\pi \times 10^{-7}}$

$= 3536776$ AT/wb.

(iv) Total reluctance $= 176838 + 250521 + 3536776$

$= 3964135$ AT/wb.

(v) Total mmf $NI = (2 \times 1000) \times 1$

$= 2000$ AT

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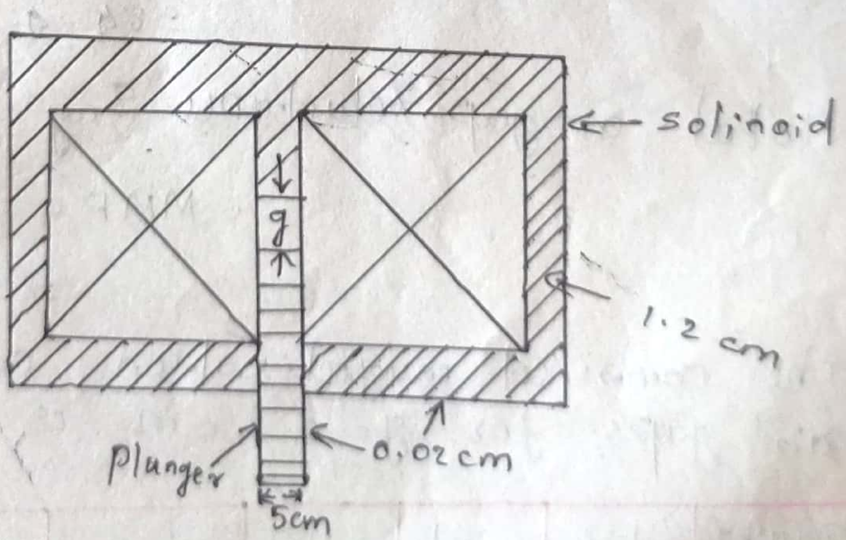
Problems

Ex Fig. shows the cross sectional view of a cylindrical iron-clad solenoid magnet. The plunger made of iron is restricted by stops to move through a limited range. The exciting coil has 1200 turns and carries a steady current of 2.25 A. The magnetising curve of the iron portion of the magnetic circuit is given below.

Flux wb	0.0010	0.00175	0.0023	0.0025	0.0026	0.00265
MMF AT	60	120	210	300	390	510



Calculate the magnetic field energy and coenergy for air gap of g = 0.2 cm and g = 1 cm with exciting current of 2.25 A in each case

Soln:-

The magnetic circuit has two air-gaps. The reluctance of each of these are calculated as follows.

Case 1: $g = 0.2$ cm.

Reluctance of the circular air-gap $= \frac{l_g}{\mu A}$

$$= \frac{0.2 \times 10^{-2}}{4\pi \times 10^{-7} \times \frac{\pi}{4} \times (0.05)^2}$$

$$= 810.5 \times 10^3$$

Reluctance of the annular air-gap

$$= \frac{0.02 \times 10^{-2}}{4\pi \times 10^{-7} \times \pi \times 0.05 \times 0.012}$$

$$= 84.4 \times 10^3$$

Total air gap reluctance $S_{ag} = 895 \times 10^3$

$$\text{MMF} = S_{ag} \times \phi$$

$$= 895 \times 10^3 \phi \text{ AT}$$

The combined magnetization curve of iron and air gaps for $g = 0.2$ cm is calculated below.

λ (WbT) $= N\phi$	1.2	2.1	2.76	3.0	3.12	3.18
AT	955	1686	2269	2538	2717	2882
i (A)	0.796	1.405	1.891	2.115	2.264	2.40

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The $\lambda - i$ curve is plotted in graph.

Field coenergy, Area oea = 3.73J

Energy area oaf = $3.11 \times 2.25 - 3.73$

$= 3.27J$

Case 2: $g = 1$ cm

Reluctance of circular air gap $= 4052.5 \times 10^3$

Reluctance of annular air gap $= 84.4 \times 10^3$

$S_{ag} = 4136.7 \times 10^3$

Because of ~~sh~~such high reluctance of air gap,

$\phi = 0.0025$ wb.

AT (air gap) = 10342

AT (iron) = 390

Even near saturation region, AT < 5% of Total AT.

$i = 2.25$ A

$MMF = 1200 \times 2.25 = 2700$ AT

$$\phi = \frac{2700}{4136.7 \times 10^3} = 0.653 \times 10^{-3} \text{ wb}$$

$\lambda = N\phi = 1200 \times 0.653 \times 10^{-3}$

$= 0.784$ wb-T

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Field energy = Coenergy = $\frac{1}{2} i \lambda$

$= \frac{1}{2} \times 2.25 \times 0.784$

$\boxed{E_F = 0.882 \text{ J}}$

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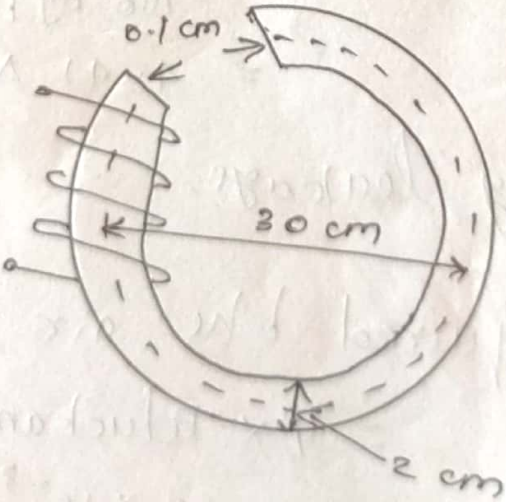
An Iron rod of 1 cm radius is bent to a ring of mean diameter 30 cm and wound with 250 turns of wire. Assume the relative permeability of iron as 800. An air gap of 0.1 cm is cut across the bent ring. Calculate the current required to produce a useful of 20,000 lines if (a) leakage is neglected and (b) leakage factor is 1.1.

$l_g = 0.1\,cm$, $d/2 = 1\,cm$

$2R_m = 30\,cm$ $N = 250$ turns , $\mu_r = 800$

$\varphi = 20,000$ lines.

$= 20,000 \times 10^{-8}$ wb.



Neglecting leakage

Total reluctance = Resistance of air gap + Reluctance of iron path.

$$R_g = \frac{l_g}{\mu_0 \mu_r a} = \frac{0.001}{4\pi \times 10^{-7} \times 1 \times \pi \times 1 \times 10^{-4}}$$

$= 2533029.59 \ A/wb$

l_i length of iron path = Total length = l_g

$$l_i = (\pi d - l_g)\ m$$

$$= \pi \times 0.3 - 0.001$$

Reluctance of iron path $= \dfrac{\pi \times 0.3 - 0.001}{4\pi \times 10^{-7} \times 800 \times \pi \times 1 \times 10^{-4}}$

$= 298[illegible] \ A/wb$

Total reluctance $= 5514018.49 \ A/wb$

MMF = Flux × Reluctance

$= 20{,}000 \times 10^{-8} \times 5514018.49$

$= 1102.8 \ AT.$

Current required $= \dfrac{1102.8}{\text{No. of turns}} = \dfrac{1102.8}{250}$

$= 4.41$ Amps.

(b) Including leakage.

MMF required the air gap portion is

$= \phi \times$ Reluctance of airgap

$= 20{,}000 \times 10^{-8} \times 2533039.59$

MMF for airgap = 506.606 Amp

For iron path the flux has to be more,

Total flux in iron path = Leakage factor × useful flux.

$= 1.1 \times 20,000 \times 10^{-8}$

$MMF_{iron} = 1.1 \times 20,000 \times 10^{-8} \times$ Reluctance of iron

$= 1.1 \times 20,000 \times 10^{-8} \times 2980988.896$

$MMF = 655.82$ Amp.

Total MMF = 1162.426 AT

$$\text{Current required} = \frac{1162.426}{250}$$

$$\boxed{\text{Current required} = 4.649\ A.}$$

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Example Two coupled coils have self and mutual inductance of $L_{11} = 2 + \frac{1}{2x}$; $L_{22} = 1 + \frac{1}{2x}$;

$L_{12} = L_{21} = \frac{1}{2x}$

over a certain range of linear displacement x. The first coil is excited by a constant current of 20 A and the second by a constant current of −10 A. Find,

(a) Mechanical work done if x changes from 0.5 to 1m.

(b) Energy supplied by each electrical source in part (a).

(c) change in field energy in part (a).

(NOV/DEC − 2018)

(Same model in April/May-2018)

Solution,

In case of current excitations, the expression of Co energy will be used.

$$W_f'(i_1, i_2, x) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$= \left(2 + \frac{1}{2x}\right) \times 200 + \frac{1}{2x} \times (-200) + \left(1 + \frac{1}{2x}\right) \times 50$$

$$= 450 + \frac{25}{x}$$

(a) $$F_f = \frac{\partial W_f'}{\partial x} = -\frac{25}{x^2}$$

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$$\Delta W_m = \int_{0.5}^{1} F_f \cdot dx = \int_{0.5}^{1} -\frac{25}{x^2} \cdot dx = -25 J$$

(b) $$\Delta W_{e_1} = \int_{\lambda_1(x=0.5)}^{\lambda_2(x=1)} i_1 \cdot d\lambda_1 = i_1 \left[\lambda_{1}(x=1) - \lambda_{1}, x=0.5\right]$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$= \left(2 + \frac{1}{2x}\right) \times 20 + \frac{1}{2x} \times (-10) = 40 + \frac{5}{x}$$

$$\lambda_1(x=0.5) = 50, \quad \lambda_1(x=1) = 45$$

$$\therefore \Delta W_{e_1} = 20(45 - 50) = -100 J$$

similarly $\Delta W_{e_2} = i_2 \left[\lambda_2(x=1) - \lambda_2(x=0.5)\right]$

$$\lambda_2 = L_{12} i_1 + L_{22} i_2$$

$$\lambda_2 = \frac{1}{2x} \times 20 + \left(1 + \frac{1}{2x}\right) \times (-10)$$

$$= -10 + \frac{5}{x}$$

$$\lambda_2(x=0.5) = 0, \quad \lambda_2(x=1) = -5$$

$$\Delta W_{e_2} = -10(-5) = 50 J$$

Net electrical energy i/p, $\Delta W_e = \Delta W_{e_1} + \Delta W_{e_2}$

$$= -100 + 50 = -50 J$$

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(c) For calculating the change in the field energy, β's have to be obtained.

$$\beta_{11} = \frac{L_{22}}{D} \; ; \; D = L_{11}L_{22} - L_{12}^2$$

$$= \frac{2x+1}{4x+3}$$

similarly, $\beta_{22} = \frac{4x+1}{4x+3}$

$$\beta_{12} = -\frac{1}{4x+3}$$

At $x = 0.5$ $\quad \beta_{11} = \frac{2}{5}, \; \beta_{22} = \frac{3}{5}, \; \beta_{12} = -\frac{1}{5}$

At $x = 1$ $\quad \beta_{11} = \frac{3}{7}, \; \beta_{22} = \frac{5}{7}, \; \beta_{12} = -1/7$

λ have been calculated at $x = 0.5, 1$ m.

Field energy is given by

$$W_f = \frac{1}{2} B_{11} \lambda_1^2 + \beta_{12} \lambda_1 \lambda_2 + \beta_{22} \lambda_2^2$$

The field energy at $x = 0.5$ m and $x = 1$ m is then calculated as,

$$W_f(x = 0.5) = \frac{1}{2} \times \frac{2}{5} \times (50)^2 = 500\,J$$

$$W_f(x=1) = \frac{1}{2} \times \frac{3}{7} \times (45)^2 - \frac{1}{7} \times 45 \times (-5) + \frac{1}{2} \times \frac{5}{7} \times (-5)^2$$

$$= 475\,J$$

Hence $\Delta W_f = W_f(x=1) - W_f(x=0.5)$

$$= 475 - 500 = -25\,J$$

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$\Delta W_f + \Delta W_m = -25 - 25 = -50 = \Delta W_e$

In the linear case with constant current excitation,

$$\Delta W_f = \Delta W_f'$$

$$W_f' = 450 + \frac{25}{x}$$

$$\Delta W_f' = W_f'(x=1) - W_f'(x=0.5)$$

$$= 475 - 500$$

$$\boxed{\Delta W_f' = -25\,J}$$

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Example: Two coils have self and mutual inductances of $L_{11} = L_{22} = \frac{2}{1+2x}$, $L_{12} = 1-2x$. The coil resistances are neglected (a) if the current I_1 is maintained at 5 A and I_2 at -2 A, Find the mechanical workdone when x increases from 0 to 0.5 m. what is the direction of the force developed? (b) when the movable part moves, calculate the energy supplied by sources supplying I_1 and I_2?

$$W_f'(i_1, i_2, x) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$= \frac{1}{2}\left(\frac{2}{1+2x}\right)5^2 + (1-2x)(-2)(5) + \frac{1}{2}\left(\frac{2}{1+2x}\right)(-2)^2$$

$$W_f' = \frac{25}{1+2x} + \frac{4}{1+2x} - 10(1-2x)$$

$$F_f' = \frac{\partial W_f'}{\partial x} = \frac{-50}{(1+2x)^2} - \frac{8}{(1+2x)^2} + 20$$

$$F_f' = 20 - \frac{58}{(1+2x)^2}$$

Mechanical workdone $= \int_{x_1} F_f' \, dx$

$$= \int_0^{0.5} 20 - \frac{58}{(1+2x)^2} \, dx$$

$$= \left[20x + 58 \frac{1}{1+2x}\right]_0^{0.5}$$

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$W = -19J$

This acts so as to reduce the distance b/w the moving part and the energising coil.

case b.

Energy supplied by the source giving I_1 is gn by,

$$W_{e1} = \int_{\lambda_{11}(0)}^{\lambda_{12}(0.5)} i_1 \, d\lambda_1$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$= \frac{2}{1+2x} i_1 + (1-2x) i_2$$

$$= \frac{2 \times 10}{1+2x} + (1-2x)(-2)$$

$$\lambda_1 = \frac{20}{1+2x} - 2(1-2x)$$

$$W_{e1} = \int_{\lambda_1(0)}^{\lambda_1(0.5)} i_1 \, d\lambda_1$$

$$= 10 \left[\lambda_1\right]_{0}^{0.5}$$

$$= 10 \left[\frac{20}{1+2x} - 2(1-2x)\right]_0^{0.5}$$

$W_{e1} = -80J$

Energy supplied by the source with I_2 is

$$W_{e2} = \int_{\lambda_2(0)}^{\lambda_2(0.5)} i_2 \cdot d\lambda_2$$

$$\lambda_2 = L_{22} i_2 + L_{12} i_1$$

$$= \frac{2}{1+2x} i_2 + (1-2x) i_1$$

$$\lambda_2 = \frac{(-4)}{1+2x} + 10(1-2x)$$

$$\lambda_2(0.5) = -2$$

$$\lambda_2(0) = 6$$

$$W_{e2} = \int_0^{0.5} (-2) [\lambda_2(0.5) - \lambda_2(0)]$$

$$= -2(-2-6)$$

$$\boxed{W_{e_2} = 16\,J}$$

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Example: The core of an electromagnet is made of an 1 cm diameter, bent into a circle of mean diameter 10 cm, a radial airgap of 1 mm being left between the ends of the rod. calculate the direct current needed in coil of 2000 turns uniformly spaced around the core to produce a magnetic flux of 0.2 mwb in the air gap. Assume that the relative permeability of the iron is 150, that the magnetic leakage factor is 1.2 and that the air gap is parallel. (April / May 2017)

Solution:-

Mean diameter $D = 10\,cm = 10 \times 10^{-2}\,m$.

$a = \pi r^2 \text{ (or) } \frac{\pi D^2}{4}$

$a = \frac{\pi}{4}(1)^2 = \pi/4 \text{ cm}^2$

$= 0.785 \times 10^{-4}\,m^2$

$l_g = 1\,mm = 1 \times 10^{-3}\,m$

$N = 2000$, $\phi_g = 0.2\,mwb$

$\mu_r = 150$, $\lambda = 1.2$

$l_i = \pi D = \pi \times 10 \times 10^{-2} = 0.3141\,m$.

$= 2\pi r$

AT for airgap. $B = \phi/a$, $B = \mu H$

$$B_g = \phi_g / a = \frac{0.2 \times 10^{-3}}{0.785 \times 10^{-4}} = 2.547\,wb/m^2$$

$$H_g = B_g/\mu_0 = \frac{2.547}{4\pi \times 10^{-7}} = 2026838\,AT/m$$

$AT_g = H_g l_g = 2026838 \times 1 \times 10^{-3}$

(mmf g) $= 2026.83\,AT.$

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At for iron path

$$\phi_i = \phi_g \times n = 0.2 \times 10^{-3} \times 1.2$$

$$= 2.4 \times 10^{-4} \text{ wb}$$

$$B_i = \frac{\phi_i}{a} = \frac{2.4 \times 10^{-4}}{0.785 \times 10^{-4}} = 3.057 \text{ wb/m}^2$$

$$H_i = \frac{B_i}{\mu_0 \mu_r} = \frac{3.057}{4\pi \times 10^{-7} \times 150} = 16217.8 \text{ AT/m}$$

$$AT_i = H_i \times l_i = 16217.8 \times 0.3141$$

$$= 5094 \text{ AT}$$

$\therefore$ Total AT_{mmf} $= AT_g + AT_i$

$$= 2026.83 + 5094$$

$$= 7120.83 \text{ AT}$$

$AT = mmf = NI$

$I = \frac{AT \text{ (or) } mmf}{N}$

$\therefore$ Direct current $I = \frac{\text{Total AT}}{N}$

$$= \frac{7120.83}{2000}$$

$$\boxed{I = 3.56 \text{ A}}$$

Example: An iron rod 1.8 cm diameter is bent to form a ring of mean diameter 25 cm and wound with 250 turns of wire. A gap of 1 mm exists in between the end faces. Calculate the current required to produce a flux of 0.6 mwb. Take relative permeability of iron as 1200. (April / May 2018)

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solution:

Mean diameter $= 25\ cm = 25 \times 10^{-2}\ m$

$N = 250$ turns

$l_g = 1\ mm = 1 \times 10^{-3}\ m$

$\varphi = 0.6\ mwb$

$\mu_r = 1200$

$\mu_0 = 4\pi \times 10^{-7}$ $\quad a = \frac{\pi d^2}{4}$ (or) πr^2

$a = \pi/4 \times (1.8)^2 = 2.544\ cm^2$ $\quad cm \times cm$

$= 2.544 \times 10^{-4}\ m^2$ $\quad 10^{-2} m \times 10^{-2} m = 10^{-4} m^2$

Length of magnetic path $l_i = \pi \times D$, $2\pi r$

$= \pi \times 25 \times 10^{-2}$

$= 0.7854\ m.$

$B = \varphi / A = \frac{0.6 \times 10^{-3}}{2.544 \times 10^{-4}}$

$B = 2.358\ wb/m^2$

At for air gap $H_g = \frac{B}{\mu_0} = \frac{2.358}{4\pi \times 10^{-7}} = 1876436.8$ AT

AT required $= H_g l_g = 1876436.8 \times 1 \times 10^{-3}$

$= 1876.43\ AT.$

At for iron part $H_i = \frac{B}{\mu_0 \mu_r} = \frac{2.358}{4\pi \times 10^{-7} \times 1200}$

$= 1563.7\ At/m$

At required

$= H_i l_i = 1563.7 \times 0.7854$

$= 1228.12\ AT$

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Total AT = 1876.43 + 1228.12 = 3104.55 A

Magnetizing current $I = \frac{3104.55}{250} = 12.42$ A

$$\boxed{I = 12.42A}$$

$$I = \frac{mmf}{N}$$

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Ex: A solenoid is wound with a coil of 200 turns. The coil is carrying a current of 1.5 A. Find the value of magnetic field intensity when the length of the coil is 80 cm.

Given:-

$N = 200$

$I = 1.5A$

$l = 80\ cm = 80 \times 10^{-2}\ m$

Solution:-

Magnetic Field intensity. $H = \frac{NI}{l}$

$= \frac{200 \times 1.5}{80 \times 10^{-2}}$

$\boxed{H = 375\ AT/m}$

Ex: A steel ring 30 cm mean diameter and a circular section 2 cm in diameter has an air gap 1 mm long. It is wound uniformly with 600 turns of wire carrying current of 2.5A. Find (1) total mmf (2) total reluctance (3) Flux. Neglect magnetic leakage. The iron path takes 40% of the total mmf.

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Given :-

steel ring mean diameter = 30 cm

No. of turns = 600

carrying current = 2.5 A

circular cross section diameter = 2 cm

solution :-

(i) Total MMF

$$\text{Total MMF} = NI = 600 \times 2.5$$

$$= 1500\ AT$$

$$MMF = 1500\ AT.$$

(ii) Total reluctance

Let M_1 be mmf for iron part

M_2 be mmf for air gap.

$$M_1 = 40\%\ \text{of}\ 1500 = 0.4 \times 1500 = 600\ AT$$

$$M_2 = 1500 - 600 = 900\ AT$$

$$M_1 = \varphi S_1 \quad \text{and} \quad M_2 = \varphi S_2$$

$$\frac{S_1}{S_2} = \frac{M_1}{M_2} = \frac{600}{900} = 0.67$$

$$S_2 = l_g / a\mu_0$$

$$a = \frac{\pi d^2}{4} = \frac{\pi 2^2}{4} = \pi\ cm^2 = \pi (1 \times 10^{-4})\ m^2$$

$$S_2 = \frac{1 \times 10^{-3}}{\pi (1 \times 10^{-4}) \times 4\pi \times 10^{-7}} = 2.53 \times 10^6\ AT/wb$$

$$\text{Total reluctance}\ S = S_1 + S_2 = 1.69 \times 10^6 + 2.53 \times 10^6$$

$$S = 4.22 \times 10^6\ AT/wb$$

(iii) Flux (φ) $\quad \varphi = \frac{\text{Total mmf}}{\text{Total reluctance}} = \frac{1500}{4.22 \times 10^6} = 0.355$ mwb.

$$\boxed{\varphi = 0.355\ mwb}$$

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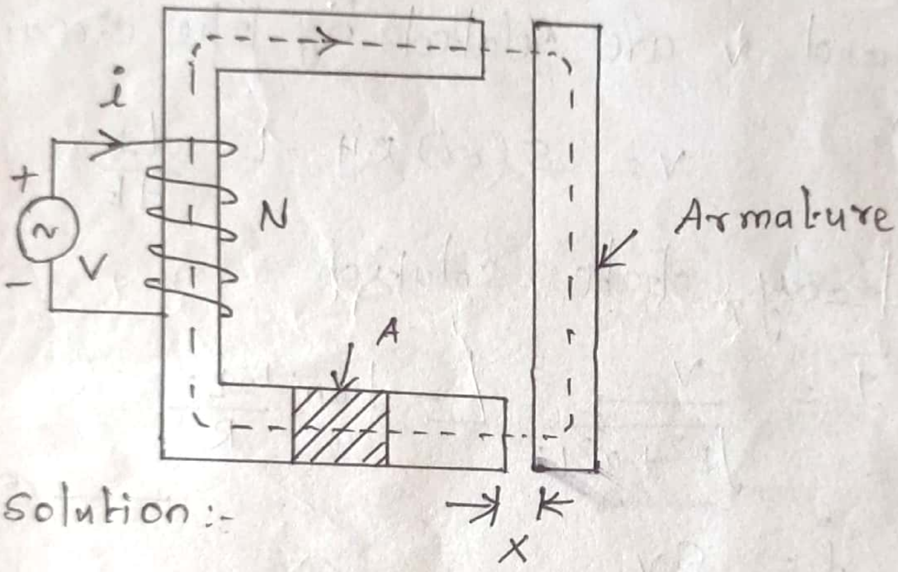
Ex: The electromagnetic relay of fig is excited from a voltage source $v = \sqrt{2}\, V \sin\omega t$.
Assuming the reluctance of the iron path of the magnetic circuit to be constant, Find the expression for the average force of the armature, when the armature is held fixed at distance x.



Solution :-

Reluctance $= \dfrac{l}{\mu A}$, $\lambda = N\varphi$

Reluctance of air path $= \dfrac{2x}{\mu_0 A} = bx$

Reluctance of iron path $= a$ (say)

Total reluctance of the magnetic path

$$S \text{ (or) } R = a + bx$$

$$W_f(\varphi, x) = \tfrac{1}{2} S \varphi^2$$

$$F_f = -\frac{\partial W_f(\varphi, x)}{\partial x}$$

$$= -\frac{1}{2}\varphi^2 \frac{\partial s}{\partial x}$$

$$= -\frac{1}{2} b\varphi^2 \quad \text{——} \ (1)$$

$\frac{\partial s}{\partial x} = b$ is +ve

so F_f is −ve

Now i and v are related by the circuit equation,

$$v = S \text{ (or) } R\,i + L\,\frac{di}{dt}$$

whose steady state solution

$$\bar{I} = \frac{V}{\sqrt{R^2+\omega^2 L^2}} \angle -\tan^{-1}\frac{\omega L}{R} \quad \text{——} \ (2)$$

Then $$i = \frac{\sqrt{2}V}{\sqrt{R^2+\omega^2L^2}} \sin\left(\omega t - \tan^{-1}\frac{\omega L}{r}\right) \quad \text{——} \ (3)$$

$$L = \frac{N^2}{S \text{ or } R}$$

Then $$\varphi = \frac{Ni}{S} = \frac{\sqrt{2}\,NV}{\sqrt{(RS)^2+(N^2\omega)^2}} \sin\left(\omega t - \tan^{-1}\frac{\omega N^2}{RS}\right) \quad \text{——} \ (4)$$

sub φ in eqn (1)

$$F_f = -\frac{bN^2V^2}{(RS)^2+(N^2\omega)^2} \sin^2\left(\omega t - \tan^{-1}\frac{N^2\omega}{RS}\right)$$

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- Time-average Force is then

$$F_f(av) = \frac{1}{T}\int_0^T F_f \cdot dt \; ; \quad T = \frac{2\pi}{\omega}$$

$$= -\frac{1}{2} \frac{bN^2 V^2}{(R s)^2 + (N^2\omega)^2}$$

Example: In the electromagnetic relay shown in fig.

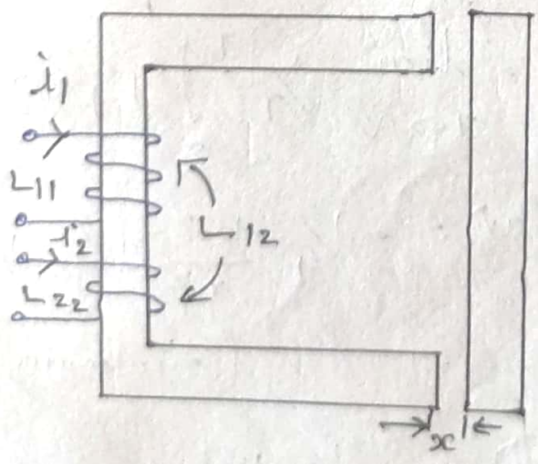
$L_{11} = k_1/x$, $L_{22} = k_2/x$, $L_{12} = k_3/x$

Find the expression for the force on the armature,

if $i_1 = I_1 \sin \omega_1 t$,

$i_2 = I_2 \sin \omega_2 t$.

Write an expression for the average force. For what relationship between ω_1 and ω_2, the average force is (i) maximum (ii) minimum.



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Solution:-

$$W_f'(i_1, i_2, x) = \frac{1}{2}\frac{k_1}{x} i_1^2 + \frac{k_2}{x} i_1 i_2 + \frac{1}{2}\frac{k_3}{x} i_2^2$$

$$F_f = \frac{\partial W_f'}{\partial x} = -\frac{1}{2}\frac{k_1}{x^2} i_1^2 - \frac{k_2}{x^2} i_1 i_2 - \frac{1}{2}\frac{k_3}{x^2} i_2^2$$

Sub for i_1, i_2

$$F_f = -\frac{1}{2}\frac{k_1}{x^2} I_1^2 \sin^2 \omega_1 t - \frac{k_2}{x^2} I_1 I_2 \sin\omega_1 t \cdot \sin\omega_2 t - \frac{1}{2}\frac{k_3}{x^2} I_2^2 \sin^2 \omega_2 t$$

$$F_f = -\frac{1}{4}\frac{k_1^2}{x^2} I_1^2 + \frac{1}{4}\frac{k_1^2}{x^2} \cos 2\omega_1 t - \frac{1}{2}\frac{k_2}{x^2} I_1 I_2 \cos(\omega_1 - \omega_2) t + \frac{1}{2}\frac{k_1}{x^2} I_1 I_2 \cos(\omega_1 + \omega_2) t - \frac{1}{4}\frac{k_2}{x^2} I_2^2 - \frac{1}{4}\frac{k_3}{x^2} I_2^2 \cos 2\omega_2 t.$$

since these are mixed frequency terms,

$$F_f(av) = \lim_{T \to \infty} \frac{1}{T} \int_0^T F_f(t) \cdot dt$$

If $\omega_1 \neq \omega_2$ $\quad F_f(av) = -\frac{1}{4}\frac{k_1^2}{x^2} I_1^2 - \frac{1}{4}\frac{k_2}{x^2} I_2^2$ (minimum Force)

If $\omega_1 = \omega_2$, $\quad F_f(av) = -\frac{1}{4}\frac{k_1^2}{x^2} I_1^2 - \frac{1}{2}\frac{k_2}{x^2} I_1 I_2 - \frac{1}{4}\frac{k_2}{x^2} I_2^2$ (maximum Force)

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UNIT - III

DC GENERATORS

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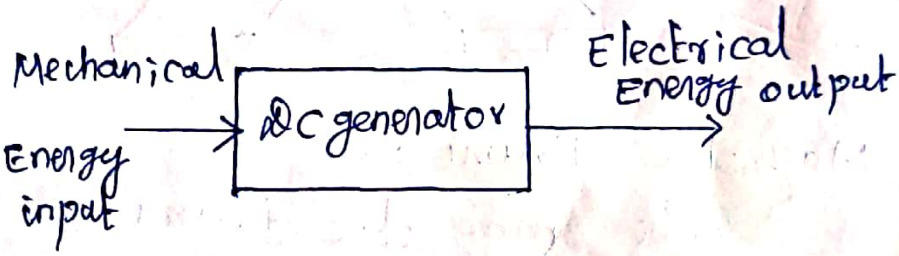
UNIT-II

DC GENERATORS.

ELECTRICAL MACHINES

With neat diagram, explain the construction & working of Dc Generator. (April/May-2018)

An electrical generator is a rotating machine which converts mechanical energy into electrical energy.



Principle:

According to Faraday's laws of electromagnetic induction, whenever a conductor is moved in a magnetic field, dynamically induced e.m.f is produced in the conductor.

Construction:

The major parts of Dc generators are

1. Magnetic frame or yoke
2. Armature
3. Poles, interpoles, windings, pole shoes
4. Commutator
5. Brushes, bearings and shaft.

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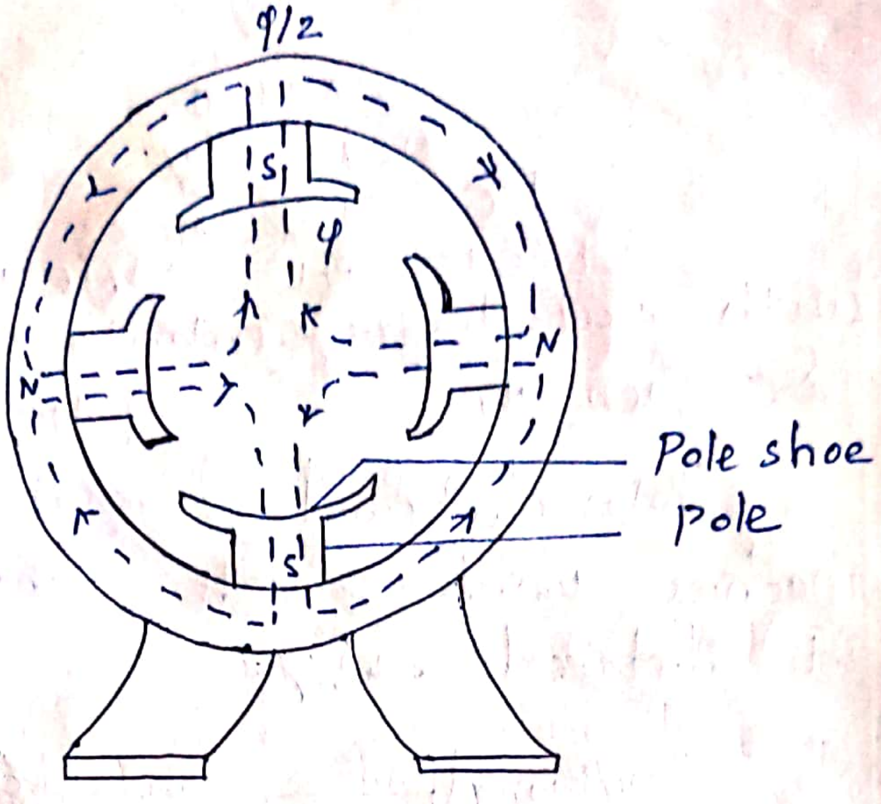

Magnetic Frame:

The magnetic frame or yoke serves two purposes.

1. It acts as a protecting cover for the whole machine and provides mechanical support for the poles.

2. It carries the magnetic flux produced by the poles.

Poles: The pole consist of (i) pole cores (ii) pole shoes and (iii) pole coils. The pole cores and pole shoes form the field magnet. Since the poles are electromagnets a field winding is wound over the pole core.

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For very small machines the poles are made up of cast iron. For larger machines cast steel is used.

To minimize eddy current losses, the pole is laminated. sheet steel laminations are used for this.

Interpoles:

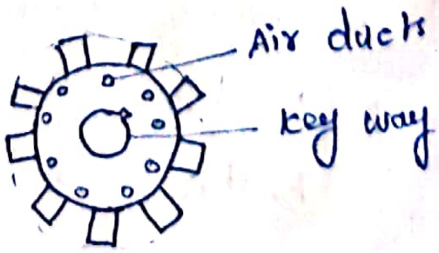
In modern dc machines commutating poles or interpoles are provided to improve commutation.

Armature:

The armature consists of an armature core and armature windings. The armature core houses the armature conductors or coils.

To reduce losses, low hysteresis steel containing a few percentage of silicon is used in the armature.

When the armature core rotates in the pole flux, eddy currents are also produced in it. To minimize the eddy current losses the armature core is laminated. The laminations are about 0.4 mm to 0.5 mm thick.



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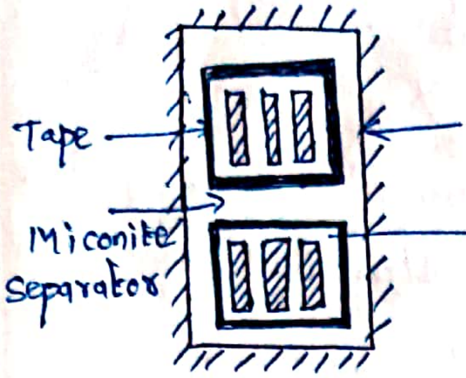

Fig. cross section of armature slots.

The armature conductors ar usually made up of copper and are housed in the slots provided in the armature. The slots are well insulated to avoid any short circuit between the armature and the conductors.

Commutator:

The commutator converts the alternating emf into unidirectional or direct emf. It is made up of wedge shaped segments or hard-drawn or drop forged copper.

Brushes and Bearings.

The brushes, which are made up of carbon or graphite, collect the current from the commutator and to convey it to the external load resistance.

EMF induced in a DC generator

Let ϕ be the flux per pole in webers.

Let P be the number of poles.

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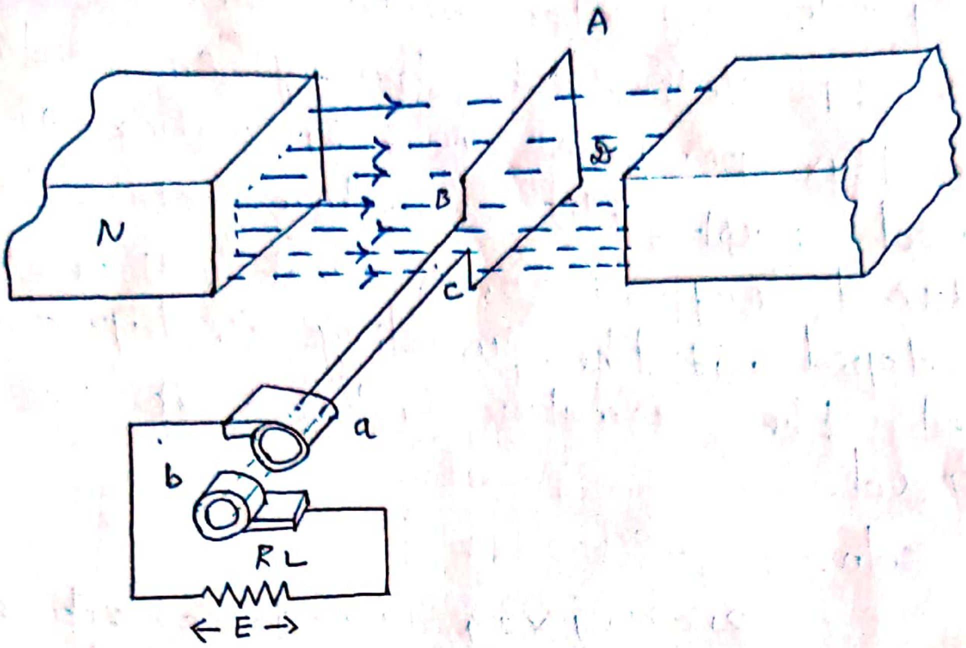

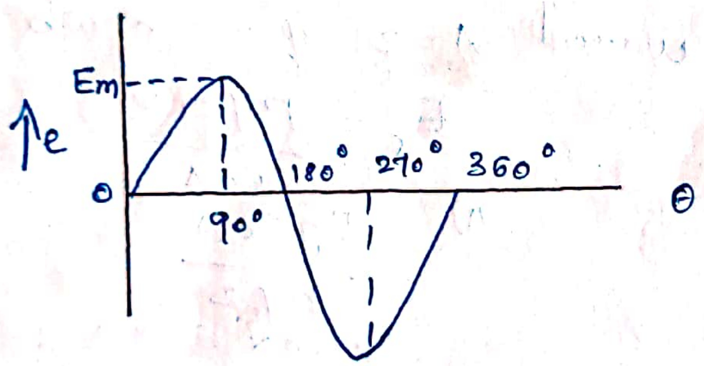

Let N be the speed of rotation in revolutions per minute.

Since the emf induced in the conductor = rate of change of flux cut.

$$e \propto \frac{d\varphi}{dt} = \frac{P\varphi}{60/N}$$

$$e = \frac{NP\varphi}{60} \text{ volts.}$$

Since there are $2/A$ conductors in series in each parallel path, the emf induced

$$\boxed{E_g = \frac{NP\varphi}{60}\,\frac{2}{A}} \text{ volts.}$$

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Ex: 1 A 4 pole generator with wave wound armature has 51 slots each having 24 conductors. The flux per pole is 0.01 weber. At what speed must the armature rotate to give an induced emf of 250V. What will be voltage developed, if the winding is lap connected and the armature rotates at the same speed.

soln:-

$Z = 51 \times 24 = 1224$ Conductors

$E_g = 250V, \quad P = 4$

For wave wound $A = 2 \quad \varphi = 0.01 wb.$

$$E_g = \frac{P\varphi Z N}{60A}$$

$$N = \frac{E_g 60 A}{P\varphi Z}$$

$$= \frac{250 \times 60 \times 2}{4 \times 0.01 \times 1224}$$

$N = 612.74$ rpm

For lap connection, $A = P$

speed $N = 612.74$ rpm

$$E_g = \frac{P\varphi Z N}{60A}$$

$$= \frac{4 \times 0.01 \times 1224 \times 612.74}{60 \times 4}$$

$$\boxed{E_g = 125V}$$

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Ex: A DC series generator delivers a load of 20kW at 400V. Its armature and series field resistances are 0.3 Ω and 0.2 Ω respectively. calculate the generated EMF and the armature current. Allow 1.1 V per brush for contact drop.

Given data:

Output power $P_{out} = 20kW$

Load voltage $V_L = 400V$

Armature resistance $R_a = 0.3\,\Omega$

series field resistance $R_{se} = 0.2\,\Omega$

Brush drop per brush $= 1.1\,V$

solution

Load current $I_L = \frac{P_{out}}{V_L} = \frac{20\times10^3}{400}$

$= 50\,A$

Here $I_a = I_L = 50\,A$

Generated emf $E_g = V_L + I_a(R_a + R_{se}) + V_{brush}$

$= 400 + 50(0.3+0.2) + 1.1\times2$

$\boxed{E_g = 427.2\,V}$

Ex: A 50kW, 250-V shunt generator operates on full load at 1500 rpm. The armature has 6 poles and is lap wound with 200 turns. Find the induced emf and the flux per pole at full load. Given that the armature and field

resistances are 0.01 and 125 Ω res
Neglect armature reaction.

Given data.

Power $P = 50\,kW$

Voltage $V = 250\,V$

Speed $N = 1500$ rpm.

Number of poles $P = 6$ with 200 turns.

Armature resistance $R_a = 0.01\,\Omega$

Field resistance $R_{sh} = 125\,\Omega$

Solution:

For a load power of 50 kW,

$$\text{Load current} = \frac{P}{V} = \frac{50\times10^3}{250}$$

$$= 200\,A$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{125} = 2\,A$$

For a shunt generator, $I_a = I_L + I_{sh}$

$$= 200 + 2$$

$$= 202\,A$$

Induced emf $E_g = V + I_a R_a = 250 + 202\times0.01$

$$\boxed{E_g = 252.02\,V}$$

Therefore Flux $\varphi = \frac{E_g \times 60}{2N} \frac{A}{P}$

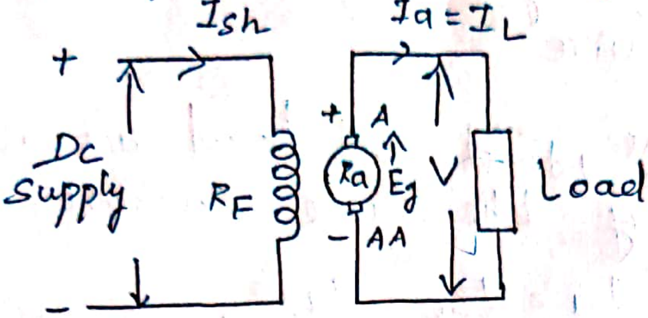
$$= \frac{252.02\times60 \times 6}{400\times1500 \times 6}$$

$$\boxed{\varphi = 25.205\,mwb}$$

Types of Dc generator.

1. separately excited Dc generators.
2. self excited Dc generators

Separately excited Dc generators.



If the field winding is excited by a separate Dc supply, then the generator is called seperately excited Dc generator.

From the above diagram

Terminal voltage $V = E_g - I_a R_a - V_{brush}$

Generated emf $E_g = V + I_a R_a + V_{brush}$

Electric power developed = $E_g I_a$

power delivered to the load = $V I_a$.

self excited Dc generator:

Types: i. series generator

ii. shunt generator

iii. Compound generator

i. series Generator.

The field winding is connected in series with the armature.

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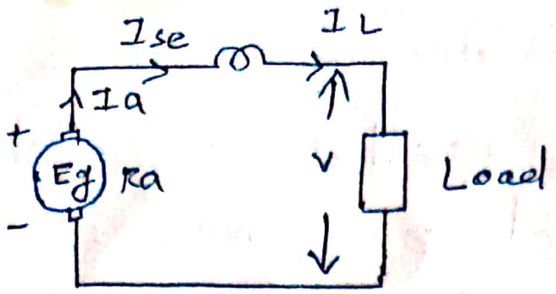


The field winding has less number of turns of thick wire.

Here, armature, field and load are all in series. so they carry the same current.

$$\therefore I_a = I_{se} = I_L$$

Generated emf

$$E_g = V + I_a R_a + I_a R_{se} + V_{brush}.$$

Terminal voltage

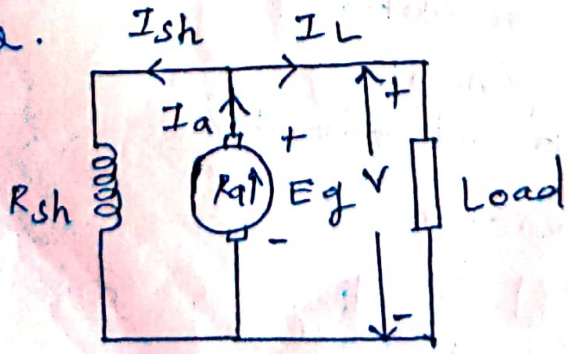
$$V = E_g - I_a R_a - I_a R_{se} - V_{brush}$$

Power developed in the armature $= E_g I_a$

power delivered to the load $= V I_L$.

(ii) Shunt generator.

In a dc shunt generator, field winding is connected across the armature. The shunt field has more number of turns. it has high resistance.



Terminal voltage $V = E_g - I_a \cdot R_a$

Shunt field current $I_{sh} = \frac{V}{R_{sh}}$

Armature current $I_a = I_L + I_{sh}$

power developed by armature $= E_g I_a$

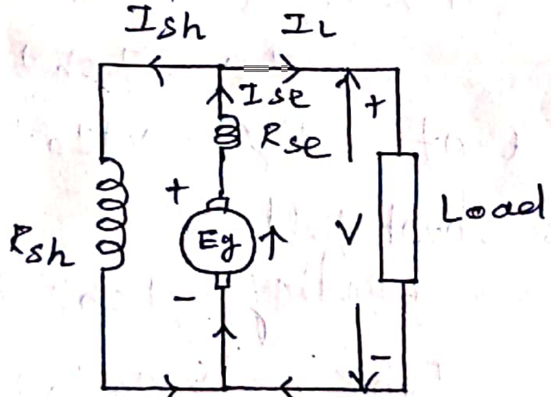
power developed to load $= V I_L$

(iii) COMPOUND GENERATOR.

1. Long shunt compound generator
2. Short shunt compound generator.

Long shunt compound generator.

Here, shunt field winding is connected across both series field and armature windings.



From the figure, series field current

$$I_{se} = I_a = I_L + I_{sh}$$

shunt field current $I_{sh} = \frac{V}{R_{sh}}$

Generated emf $E_g = V + I_a(R_a + R_{se}) + V_{brush}$

Terminal voltage $V = E_g - I_a(R_a + R_{se}) - V_{brush}$

power developed in Armature $= E_g I_a$

power delivered to load $= V I_L$

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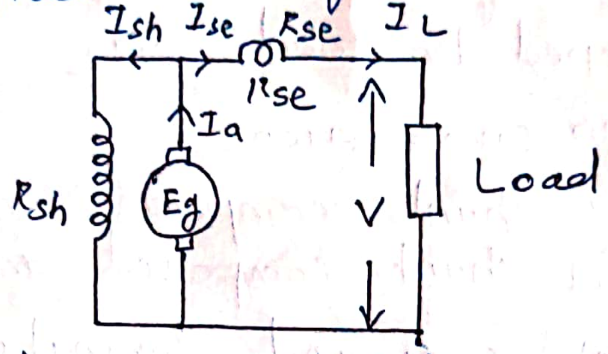
Short Shunt Compound Generator.

Here, Shunt field winding is connec
in parallel with the armature and this
Combination is connected in series with
series field winding.



From the figure

series field current $= I_{se} = I_L$

Load current $I_L = I_{se}$

$I_a = I_{sh} + I_{se}$

Generated emf. $E_g = V + I_a \cdot R_a + I_{se} R_{se} + V_{brush}$

Voltage across shunt field winding $= I_{sh} R_{sh}$

$$I_{sh} R_{sh} = V + I_a \cdot R_a + I_{se} R_{se} + V_{brush} - I_a \cdot R_a - V_{brush}$$

$$= V + I_{se} R_{se}$$

shunt field current $= \dfrac{V + I_{se} \cdot R_{se}}{R_{sh}}$

Terminal voltage $= E_g - I_a \cdot R_a - I_{se} R_{se} - V_{brush}$

power developed in armature $= E_g \cdot I_a$

power delivered to load $= V I_L$.

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A 4 pole lap connected shunt generator has $R_{sh} = 100\,\Omega$ and $R_a = 0.1\,\Omega$ and supplies sixty lamps each rated 40 W, 200V. calculate the armature ct, induced emf and current in each parallel path of the armature. Allow a brush drop of 1V per brush.

$$P_O = 60 \times 40 = 2400 \text{ watts.}$$

Load current $= P_o/V$

$$I_L = \frac{2400}{200} = 12\text{ A}$$

Field ct $I_{sh} = \frac{V}{R_{sh}} = \frac{200}{100} = 2\text{ A}$

$$I_a = I_L + I_{sh} = 12 + 2 = 14\text{ A}$$

ct per path $= 14/4 = 3.5\text{ A}$

Induced emf $= V + I_a R_a + V_{brush}$.

$$= 200 + 14 \times 0.1 + 2 \times 1$$

$$\boxed{E_g = 203.4 \text{ volts.}}$$

2. A 10 pole DC shunt generator with 800 wave connected conductors and running at 600 rpm supplies a load of $15\,\Omega$ resistance at a terminal vge of 240V. The armature resistance is $0.28\,\Omega$ and field resistance is $240\,\Omega$. Determine

the armature ct, the induced and flux per pole.

soln:-

$$I_L = \frac{V}{R_L} = \frac{240}{15}$$

$$I_L = 16\,A$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{240}{240} = 1\,A$$

$$I_a = I_L + I_{sh}$$

$$= 16 + 1 = 17\,A$$

$$E_g = V + I_a \cdot R_a$$

$$= 240 + 17 \times 0.28$$

$$E_g = 244.76\,V$$

$$E_g = \frac{\phi Z N P}{60\,A}$$

$$= 6.12 \times 10^{-3}\,wb$$

$$\boxed{\phi = 6.12\,mwb}$$

3. A seperately excited dc generator running at 1000 rpm supplied 110 A at 220V a resistive load. If the load resistance remain constant, what will be the load ct, if the speed to 800 rpm? Field current is unaltered, assume a voltage drop 1 V per brush. Ignore the effect of armature reaction.

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$$R_L = \frac{V}{I} = \frac{220}{110} = 2\,\Omega$$

$$E_{g1} = V_1 + I_{a1} R_a + V_{brush}$$

$$= 220 + 110 \times 0.02 + (1 \times 2)$$

$$= 224.2\ V$$

At 800 rpm, $E_{g2}/E_{g1} = \frac{N_2}{N_1}$

$$E_{g2} = 224.2 \times \frac{800}{1000}$$

$$E_{g2} = 179.36\ V$$

$$V_2 = E_{g2} - I_{a2} \cdot R_a - V_{brush}$$

$$= 179.36 - 0.02 \times I_{a2} - 2$$

$$= 177.36 - 0.02 I_{a2}$$

$$I_{a2} = \frac{V_2}{R_L}$$

$$= \frac{177.36 - 0.02 I_{a2}}{2}$$

$$\boxed{I_{a2} = 87.8 A}$$

Ex4. A DC series generator delivers a load of 20 kW at 400V. Its armature and series field resistances are $0.3\,\Omega$ and $0.2\,\Omega$ respectively. Calculate the generated emf and the armature ct. Allow 1.1V per brush for contact drop.

$$I_L = \frac{P_{out}}{V_L} = 50\ A$$

$I_a = I_L = 50\ A$

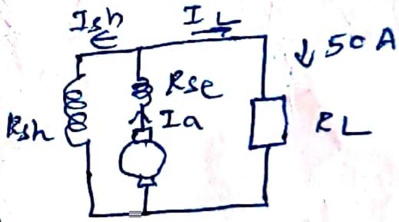
$E_g = V_L + I_a(R_a + R_{se}) + V_{brush}$

$= 400 + 50(0.3 + 0.2) + 1.1 \times 2$

$\boxed{E_g = 427.2\ V}$

Ex5. A compound generator delivers a load of 50 A at 500V. The resistance are $R_a = 0.05\ \Omega$, $R_{se} = 0.03\ \Omega$ and $R_{sh} = 250\ \Omega$. Find the induced emf, if contact drop is 1V per brush. Neglect armature reaction. Assume (a) long shunt (b) short shunt.

For long shunt



$V = 500\ V$

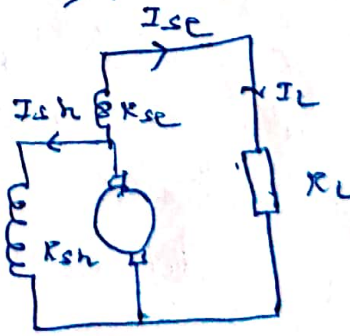
$I_{sh} = \frac{V}{R_{sh}} = \frac{500}{250} = 2\ A$

$I_a = I_L + I_{sh} = 52\ A.$

$E_g = V + I_a(R_{se} + R_a) + V_{brush}$

$= 500 + 52(0.03 + 0.05) + 2 \times 1$

$\boxed{E_g = 506.16\ V}$



$I_{sh} = \frac{500 + 50 \times 0.3}{250}$

$= 2.006\ A$

$I_a = I_L + I_{sh} = 50 + 2.006 = 52.006\ A.$

$E_g = V + I_a R_a + I_{se} R_{se} + V_{brush}$

$= 500 + 52.006 \times 0.05 + 50 \times 0.03 + 2$

$\boxed{E_g = 506.1003\ V}$

Characteristics of DC Generator.

There are three types

1. Open circuit characteristics (OCC) - (E_g Vs I_f)
2. Internal characteristics or total characteristics - (E Vs I_a)
3. External (or) voltage regulated characteristics - (V Vs I_L)

Seperately excited DC generator characteristics.

For a given DC generator, the induced emf is proportional to the flux and the speed. If speed is kept constant, and the flux is varied, the induced emf also varies.

Since $E_g = \dfrac{P\varphi Z N}{60A}$

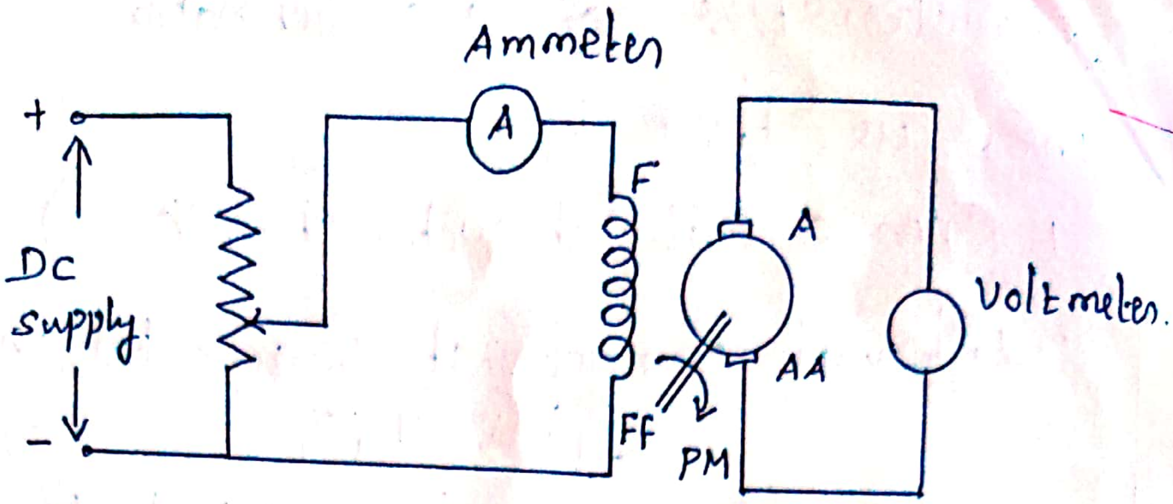
$$E_g \propto \varphi N$$

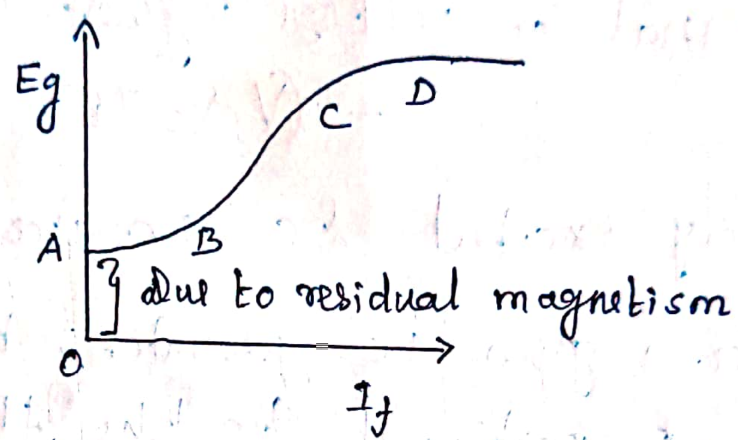
$$N = \text{constant}$$

$$\phi \uparrow ;\ E_g \uparrow$$

→ The variation of flux with the induced emf is called the no load magnetisation curve or OCC of the generator.

→





From fig. OA - Residual magnetism when the field current is zero.

Internal characteristics.

This curve is drawn between emf E and armature current I_a.

$I_a \uparrow$ $E \downarrow$ due to armature reaction.

External characteristics.

This curve is drawn between the terminal voltage V and armature current I_a

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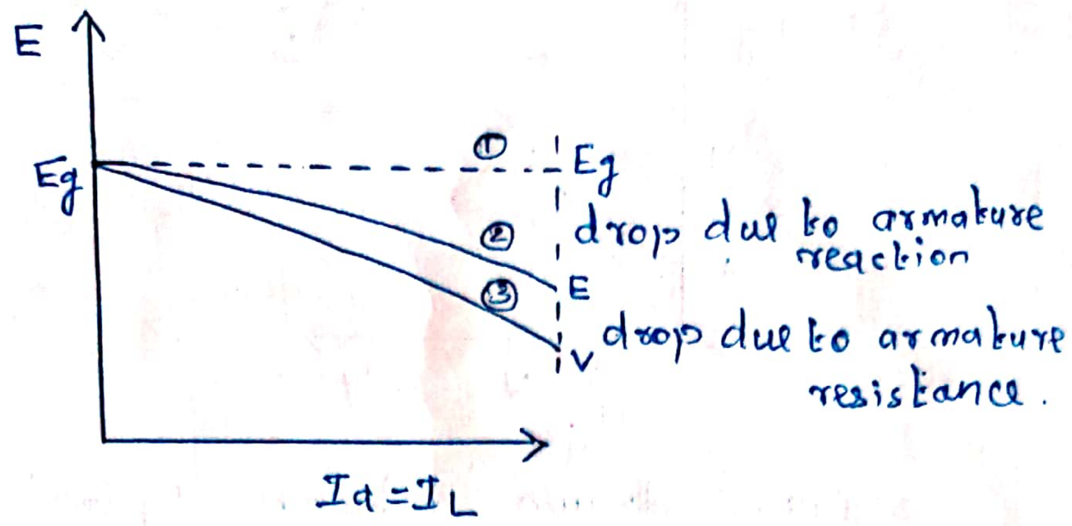
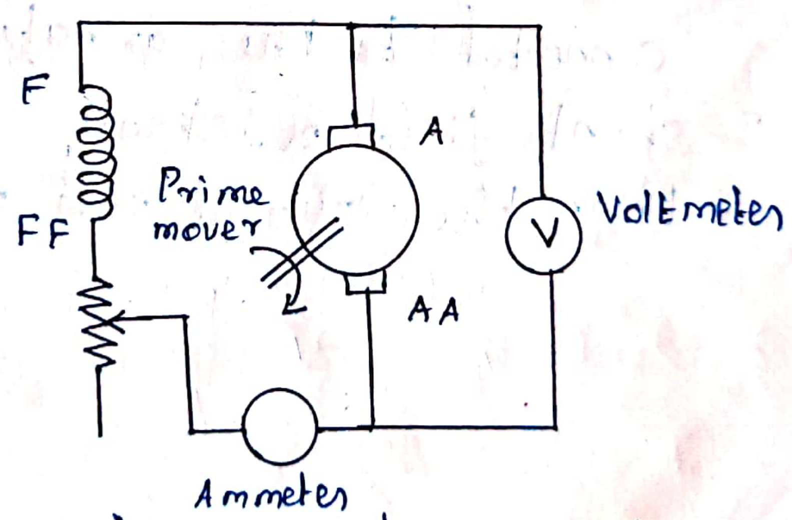


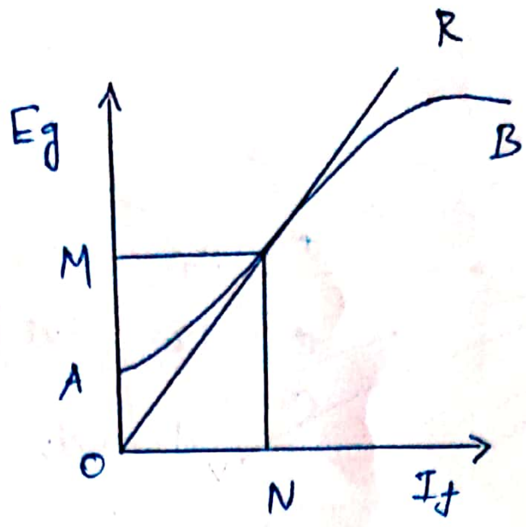
I_L (or) I_a ↑ , E ↓ due to armature resistance.

DC shunt generator characteristics.

In this induced emf will depend on the field current and at the same time the field current will depend on the induced emf.



→ Here generator speed is constant.
→ Initially field current is zero.
→ emf is induced in the generator due to residual magnetism.



→ Curve drawn between I_f and E_g.

Critical resistance:

$$R_C = \frac{OM}{ON}$$

Conditions for build up of a self excited shunt generator.

1. There must be some residual magnetism.
2. Shunt field coils should be properly connected to the armature terminals.
3. Shunt field resistance should be less than the critical resistance.

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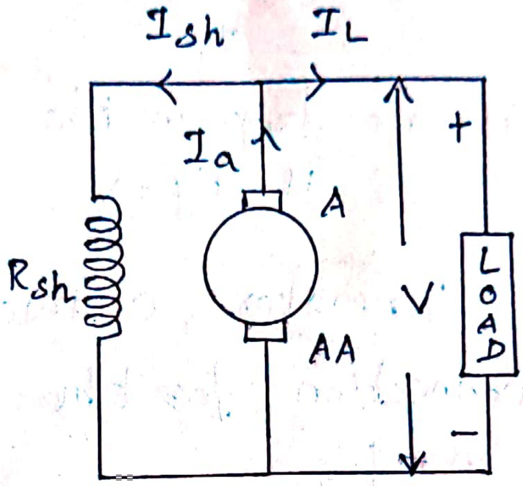
Internal and External characteristics (or) Load characteristics.

Fig shows the connections for a d.c shunt generator.



$I_a = I_{sh} + I_L$, $I_L \uparrow$, $I_a \uparrow$

E_g = constant but terminal voltage V reduces.

Reason for V reduces:-

i. Drop on Ra

ii. Brush contact drop.

iii. Drop due to armature reaction.

$$V = E_g - I_a \cdot R_a$$

Fig shows internal and external characteristics of dc shunt generator.

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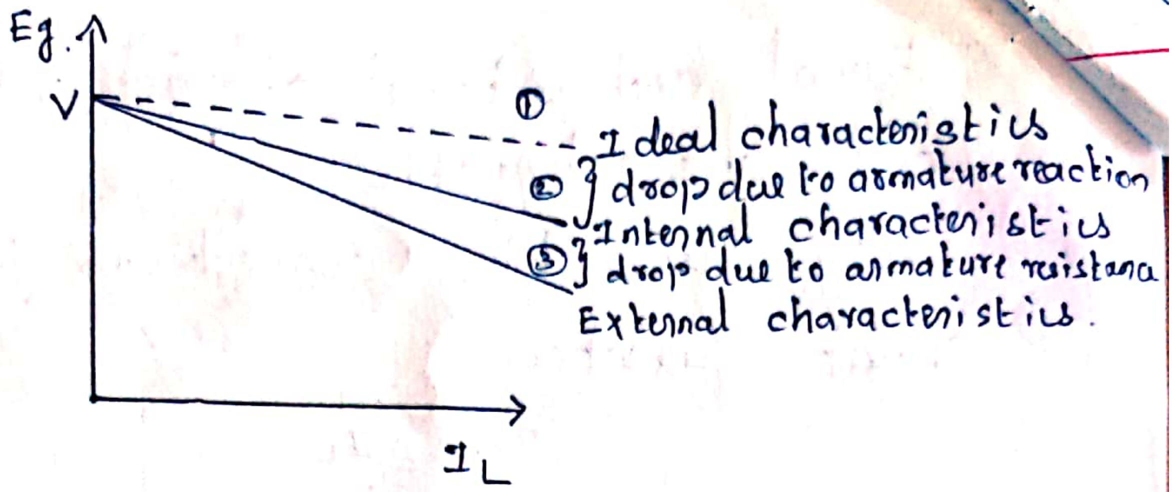
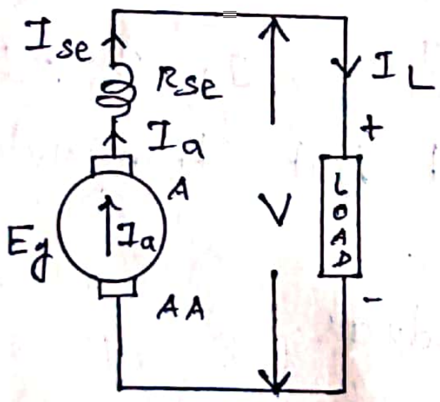


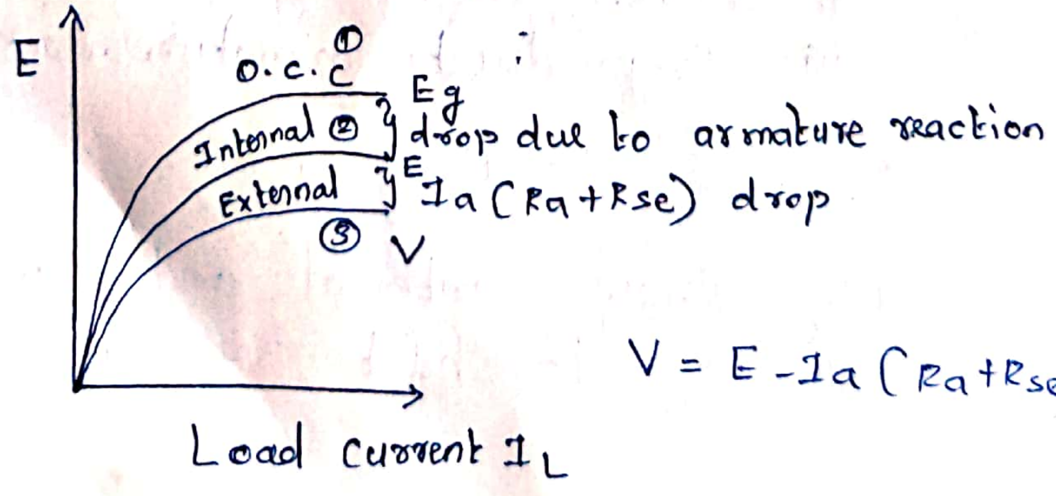
there is no drop in the armature
ie $E_g = V$.

DC Series Generator characteristics.

The connection for the DC series generator is shown in fig.



$$I_a = I_{se} = I_L$$



$$V = E - I_a(R_a + R_{se})$$

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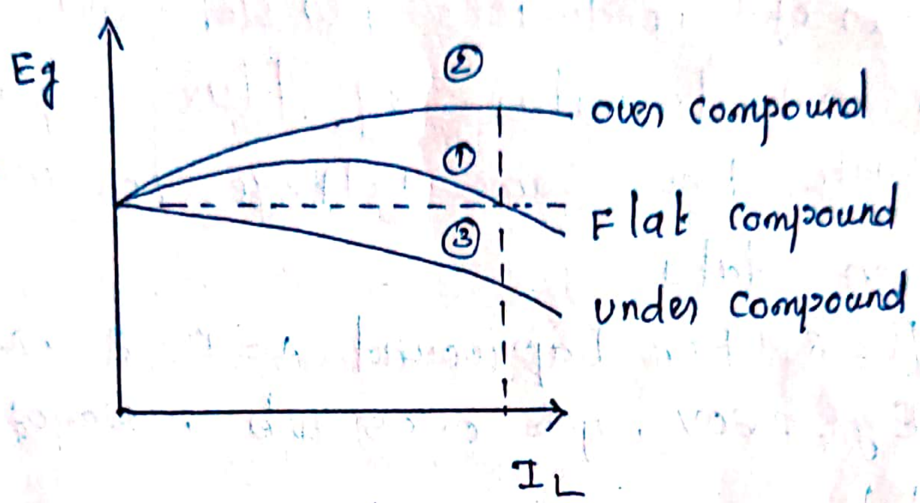
Compound Generator.

Fig shows the external characteristics of compound generator.



For Flat compound $E_g = V$

For over compound $V > E_g$

For under Compound $V < E_g$.

Applications

- supplying nearly constant load
- special electrical application where DC equipment.
- series generators are used in boosters.
- Compound generators are Used in constancy of voltage required.

Example: A 8 pole lap wound armature rotated at 350 rpm is required to generate 260 V. The useful flux per pole is about 0.05 wb. If the armature has 120 slots, calculate number of conductors per slot and hence determine the actual value of flux required to generate the same voltage for wave wound.

Given data

$P = 8$, For Lap wound $A = P = 8$, $N = 350$ rpm,

$E_g = 260V$, $\varphi = 0.05$ wb, No. of slots = 120.

Solution:

Generated $E_g = \dfrac{P\varphi Z N}{60A}$

$$Z = \frac{E_g\, 60A}{P\varphi N} = \frac{260\times60\times8}{8\times0.05\times350} = 891$$

$\therefore$ No. of conductors/slot $= \dfrac{Z}{\text{No. of slots}}$

$= 891/120 = 7.425$

For wave wound machine

$A = 2$

$$E_g = \frac{P\varphi Z N}{60A}$$

$$\varphi = \frac{E_g\, 60A}{P Z N} = \frac{260\times60\times2}{8\times891\times350}$$

$$\boxed{\varphi = 0.0125 \text{ wb}}$$

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2. A 220 V dc generator supplies 4 kW at a terminal voltage of 220 V, the armature resistance being 0.4 Ω. If the machine is now operated as a motor at the same terminal voltage with the same armature current, calculate the ratio of generator speed to motor speed. Assume that the flux/pole is made to increase by 10% as the operation is changed over from generator to motor. (NOV/DEC 2018)

Solution:

$$n \propto \frac{E_a}{\varphi}$$

As a generator $I_a = \frac{P}{E_a} = \frac{4 \times 1000}{220}$

$$I_a = 18.18\ A$$

$E_{ag} = V + I_a \cdot R_a$, $E_{ag} = 220 + 18.18 \times 0.4$

$$E_{ag} = 227.3\ V$$

As a motor $E_{am} = V - I_a \cdot R_a$

$$= 220 - 18.18 \times 0.4$$

$$= 212.7\ V$$

$$\frac{N_g}{N_m} = \frac{E_{ag}}{E_{am}} \times \frac{\varphi_m}{\varphi_g}$$

$$= \frac{227.3}{212.7} \times 1.11$$

$$\frac{N_g}{N_m} = 1.176.$$

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Example: A dc shunt generator driven by a belt from an engine runs at 750 rpm while feeding 100 kW of electric power on to 230 V mains. When the belt breaks it contineous to run as a motor drawing 9 kW from the mains. At what speed would it run?

Given armature resistance 0.08 Ω and field resistance 115 Ω.

Soln:-

Field current $I_{sh} = \frac{V}{R_{sh}} = \frac{230}{115} = 2A.$

As a generator $I_L = \frac{P}{V} = \frac{100 \times 10^3}{230} = 434.8A$

$I_f = 2A$

$I_a = I_L + I_f$

$= 434.8 + 2$

$\boxed{I_a = 436.8 \ A}$

$E_{ag} = V + I_a \cdot R_a$

$= 230 + 0.08 \times 436.8$

$= 264.9 V$

$N_g = 750 rpm.$

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As a motor,

$$I_L = \frac{P_m}{V}$$

$$= \frac{9 \times 10^3}{230}$$

$$I_L = 39.13 \text{ A} \quad , \quad I_f = 2 \text{ A}$$

$$I_a = I_L - I_f = 39.13 - 2 = 37.13 \text{ A}$$

$$E_{am} = V - I_a \cdot R_a$$

$$= 230 - 37.13 \times 0.08$$

$$\boxed{E_{am} = 227 \text{ V}}$$

The induced emf E_a is proportional to armature speed,

$$\frac{N_{motor}}{N_{generator}} = \frac{227}{264.9}$$

$$N_{motor} = \frac{227}{264.9} \times 750$$

$$\boxed{N_{motor} = 642.7 \text{ rpm}}$$

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Difference between lap winding and wave winding.

Lap winding	Wave winding.
1. Coil span $y_{cs} = \frac{S}{P}$ (lower integer)	$y_{cs} = \frac{S}{P}$ lower integer.
2. Back pitch $y_b = Uy_{cs} \pm 1$	$y_b = Uy_{cs} + 1$
3. Commutator pitch $y_c = \pm 1$	$y_c = \frac{2(C \pm 1)}{P}$
4. Front pitch $y_f = y_b \pm 2$	$y_f = 2y_c - y_b$
5. Parallel paths, $A = P$	$A = 2$
Conductor current $I_c = I_a / A$	$I_c = I_a / 2$
6. Number of brushes $A = P$	Number of brush $= 2$
7. No dummy coil needed	Dummy coil may be needed.
8. Equilizer ring needed	Equilizer ring not needed.

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(28)

A DC shunt generator driven by a belt from an engine runs at 750 rpm while feeding 100 kW of electric power into 230 V mains. When the belt breaks it contineous to run as a motor drawing 9 kW from the mains. At what speed would it run? Given armature resistance 0.08 Ω and field resistance 115 Ω

Note: In a shunt machine the field is connected across the armature and is also connected directly to the 230 V mains. The field excitation therefore remains constant as the machine operation changes as described above. Nov/Dec 2018.

Gin data

Generator $N = 750$ rpm | $R_a = 0.08\,\Omega$

$Power = 100$ kW | $R_{sh} = 115\,\Omega$

$V = 230$ V

motor $Power = 9$ kW

$V = 230$ V

Soln:-

$$I_L = \frac{100 \times 10^3}{230} = 434.78 \text{ A}$$

$$I_{sh} = \frac{230}{115} = 2 \text{ A}$$

$$I_a = I_L + I_{sh} = 436.78 \text{ A}$$

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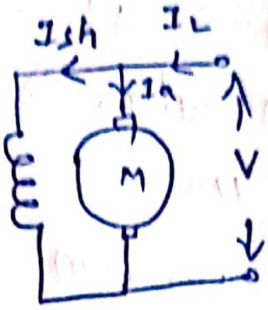
$$E_g = V + I_a . R_a$$

$$= 230 + 436.78 \times 0.08$$

$$\boxed{E_{g_1} = 264.94 \text{ V}}$$

$$I_L = I_a + I_{sh}$$



$$I_{sh} = \frac{V}{R_{sh}}$$

$$= \frac{230}{115} = 2 \text{ A}$$

$$I_L = \frac{P}{V} = \frac{9 \times 10^3}{230} = 39.130$$

$$I_L = 39.130 \text{ A}$$

$$I_a = I_L - I_{sh} = 39.130 - 2 = 37.130 \text{ A}$$

$$E_g = V - I_a . R_a$$

$$= 230 - 37.130 \times 0.08$$

$$= 230 - 2.97$$

$$E_{g_2} = 227.0296 \text{ V}$$

$$\frac{E_{g_2}}{E_{g_1}} = \frac{N_2}{N_1}$$

$$N_2 = N_1 \times \frac{E_{g_2}}{E_{g_1}} = 750 \times \frac{227.0296}{264.94}$$

$$= 643.436$$

$$\boxed{N_2 = 643 \text{ rpm}}$$

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A 4 pole, lap wound, dc generator has 42 coils with 8 turns per coils. It is driven at 1120 rpm. If useful flux per pole is 21 mWb, calculate the generated emf. Find the speed at which it is to be driven to generate the same emf as calculated above with wave wound armature. (April / May 2019)

Gn data

$P = 4$, Lap wound

$Z = 42 \times 8 = 336$

$N = 1120$ rpm.

$\varphi = 21$ mWb.

Soln:-

1. Generated emf $E_g = \dfrac{\varphi Z N P}{60 A}$

$$= \frac{21 \times 10^{-3} \times 336 \times 1120 \times 4}{60 \times 4}$$

$$E_g = 131.712 \text{ V}$$

2. For wave connected $A = 2$

$$E_g = 131.712 \text{ V}$$

$$N = \frac{E_g \times 60 A}{\varphi Z P} = \frac{131.712 \times 60 \times 2}{21 \times 10^{-3} \times 336 \times 4}$$

$$\boxed{N = 560 \text{ rpm}}$$

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Two 500V DC shunt generators rated at 100 kW and 200 kW respectively are operating in parallel. Both of them have linearly drooping external characteristics. Voltage regulation of the first generator is 4% and that of the second generator is 6%. Determine the common bus voltage and current shared by each of the generators when their parallel combination is to supply a current of 300A.

April / May 2018

Soln:- For 100 kW generator →

Full load voltage drop $= 500 \times 0.04$

$= 20\,V$

Full load current $= \frac{100 \times 10^3}{500} = 200\,A$

Drop per ampere $= \frac{20}{200} = 0.1\,V/A$

For 200 kW generator →

Full load voltage drop $= 500 \times 0.06$

$= 30\,V$

Full load current $= \frac{200 \times 10^3}{500} = 400\,A$

Drop per ampere $= \frac{30}{400} = 0.075\,V/A$

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(i)

If I_1 and I_2 are currents supplied by the two shunt generators and V the terminal voltage, then

$$V = 500 - 0.1 I_1$$

$$V = 500 - 0.075 I_2$$

$$500 - 0.1 I_1 = 500 - 0.075 I_2$$

$$0.1 I_1 = 0.075 I_2$$

$$I_1 = 0.75 I_2$$

$$I_1 + I_2 = 300\ A$$

$$0.75 I_2 + I_2 = 300\ A$$

$$1.75 I_2 = 300$$

$$I_2 = \frac{300}{1.75}$$

$$\boxed{\begin{aligned} I_2 &= 171.428\ A \\ I_1 &= 128.571\ A \end{aligned}}$$

(ii) Terminal voltage $V = 500 - 0.1 \times I_1$

$$= 500 - 0.1 \times 128.571$$

$$\boxed{V = 487.143\ V}$$

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A 4 pole, 50 kW, 250V, wave wound Shunt generator has 400 armature conductors. Brushes are given a lead of 4 commutator segments. Calculate the demagnetization ampere-turns per pole if shunt field resistance is 50Ω. Also calculate extra shunt field turns per pole to neutralize the demagnetization.

(April/May 2018)

Gn data

$P = 4$

Power $= 50\,kW$

$V = 250\,V$, wave wound

$Z = 400$

Commutator segments $= 4$

$R_{sh} = 50\Omega$

Soln:- $I_L = \frac{50 \times 10^3}{250} = 200\,A$

$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{50} = 5A$

Armature current $I_a = I_L + I_{sh}$

$= 200 + 5 = 205\,A$

Current in each conductor $I = I_a/A = \frac{205}{2} = 102.5\,A$

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No. of commutator segments $= \frac{Z}{A} = \frac{400}{2} = 200$

$$\theta m = \frac{4}{360} \times 360^\circ = 4^\circ$$

Demagnetizing amp-turns/pole $AT_d/\text{pole} = \frac{ZI\,\theta m}{360}$

$$= 400 \times 102.5 \times \frac{3}{360}$$

$$\boxed{AT_d/\text{pole} = 341.666}$$

Extra shunt turns/pole $= \frac{AT_d}{I_{sh}} = \frac{341.666}{5}$

$$= 68.333$$

$$\boxed{\text{Extra Turns/pole} \simeq 68}$$

A 4 pole lap wound shunt generator supplies 60 lamps of 100 W, 240 V each. The field and armature resistances are 55 Ω and 0.18 Ω respectively. If the brush drop is 1V for each brush find (i) Armature current (ii) Current per path (iii) Generated emf (iv) Power o/p of dc machine. (April/May-2017)

Gr data. $P = 4$, Lap wound

Power $= 60 \times 100 = 6000$ watts.

$V = 240\,V$, $R_{sh} = 55\,\Omega$, $R_a = 0.18\,\Omega$

$V_{brush} = 1V$.

$$I_L = \frac{P}{V_L}$$

$$= \frac{60,000}{240} = 250\ A$$

$$I_a = I_L + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{240}{55} = 4.36\ A$$

$$I_a = 250 + 4.36 = 254.36\ A$$

$$\boxed{I_a = 254.36\ A}$$

(ii) current per path $= \frac{254.36}{4} = 63.59$

$[\because (I_a/A)$

For Lap $A = P]$

(iii) Generated emf $E_g = V + I_a R_a + V_{brush}$

$$= 240 + 254.36 \times 0.18 + 1 \times 2$$

$$\boxed{E_g = 287.78\ V.}$$

(iv) P_{out} in armature $= E_g I_a$

$$= 287.78 \times 254.36$$

$$\boxed{P_{out} = 73.199\ k\ Watts.}$$

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Armature Reaction:

→ The term armature reaction means the effect of the mmf set up by the armature current on the distribution of mmf under main poles of a DC machine.

→ The main field flux gets weakened (or) gets demagnetized. This effect is called Demagnetization effect.

→ The main field flux gets distorted. This effect is called cross-magnetisation effect.

Main Field of the DC Machine:

→ When the DC machine is excited, under no load, the main field originates.

→ This field is assumed to be develop magnetic lines of force when no current flows in the armature conductor. This is indicated as ϕm.

→ The main field is found to be symmetrically distributed with respect to polar axis.

→ The magnetic neutral axis (MNA) is considered to be coinsiding with the geometric neutral axis. (GNA).

→ Polar axis lies along the centre of the main pole. This is called direct axis (d-axis).

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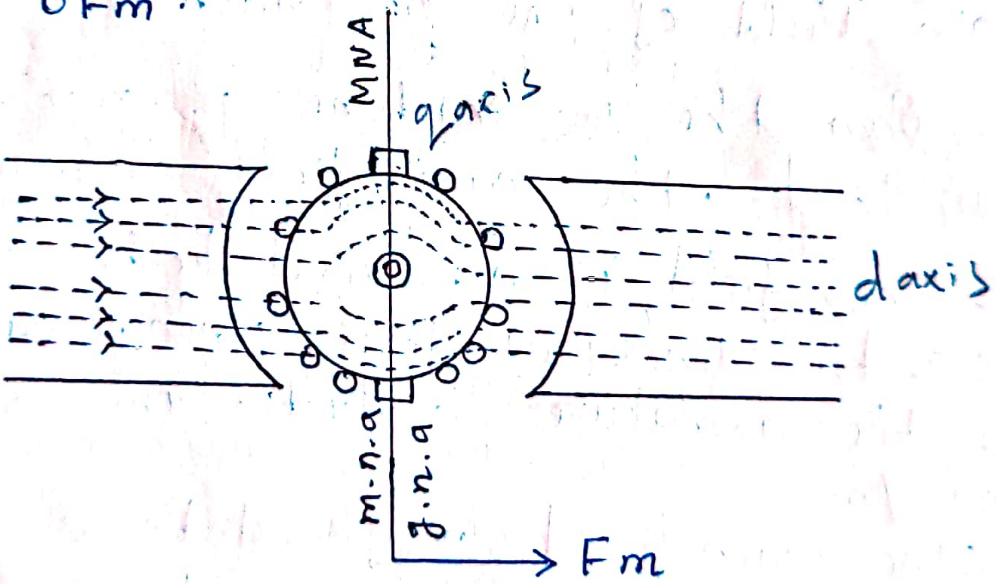
→ The axis which is at 90° to the direct axis is called the quadrature axis. (q-axis).

→ At this axis, the armature coil is parallel to the flux lines and hence the induced emf becomes zero. $\left(\frac{d\phi}{dt} = 0\right)$

→ MNA is called axis of commutation because the current reversal in the armature conductor takes place across this axis.

→ Field vector is indicated by the vector OFm.



Armature Field of the DC machine.

→ When a DC generator is loaded, the armature current flows through the armature conductors, direction is found by applying flemmings right hand rule.

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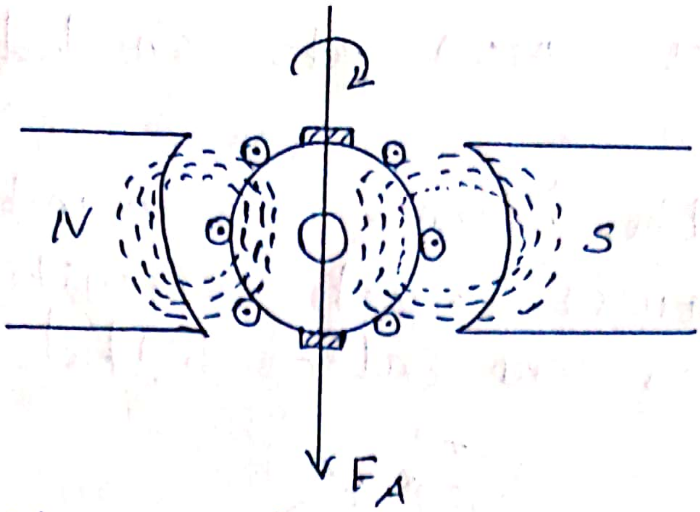

→ If the generator is assumed to be under excited condition, the armature mmf alone acts upon the air gap.

→ The mmf of the armature conductor is combine to send the flux downwards through the armature, the vector F_A parallel to brush axis.

Interaction between main field and armature field mmf.

→ when both the fluxes act simultaneously the armature field mmf and the main field mmf interact with each other.

→ As a result, the air gap flux becomes distorted.

→ Resultant flux is crowded at trailing pole tips, weakened at leading pole tips.

→ This results in non uniform distribution of flux density along the air gaps.

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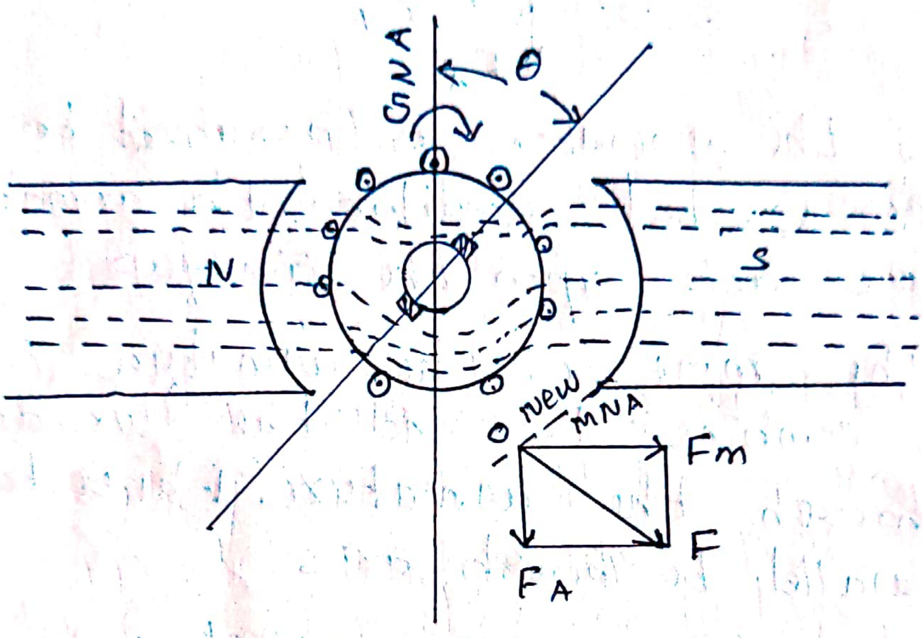
→ Hence MNA gets shifted from GNA by θ.

→ If the brushes are in initial condition it causes short circuiting, and the brushes are also shifted.



Cross magnetisation :-

→ As a result of shift in position of MNA, the armature flux strengthens each main pole at one end of weakens the pole at other end.

→ The components acts at 90° to the main field flux and hence causes distortion in the main flux.

→ This component is called cross magnetisation flux.

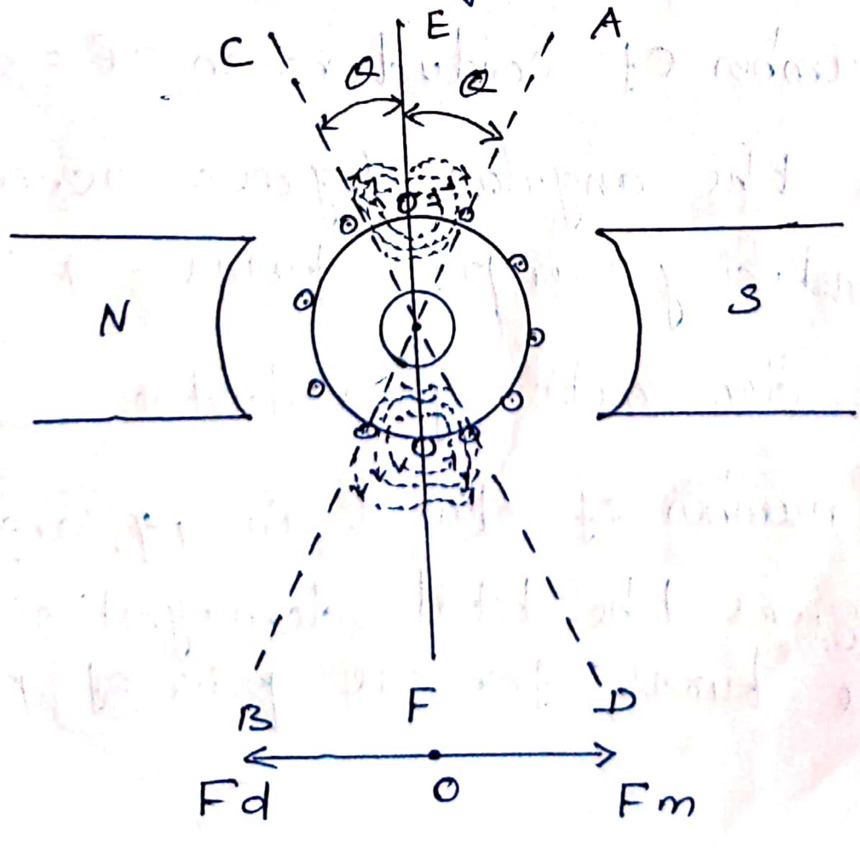
Demagnetisation :-

→ If the iron part in the magnetic circuit to remain saturation, the non uniform distribution of the flux density results in a net reduction in the flux per pole where the magnitude of reduction depends upon the state of magnetization. This is demagnetisation effect of the armature reaction.

Armature conductors and ampere-turns.

Demagnetizing MMF (AT_d)

→ The current flows in the direction opposite to the main field flux and hence the corresponding conductors are termed as Demagnetising armature conductor.



Q8

→ Assuming two conductors constitute one turns, the demagnetising ampere turns can be calculated as follows.

Z → Total number of armature conductors.

I → Current in each armature conductor

I_a → Armature current.

$I_a/2 = I$ for wave winding

$I_a/A = I$ for Lap winding.

A = number of parallel path

φ = Forwarded lead angle of the MNA.

Mechanical degree converted by all the Z conductor $= 360°$

∴ Number of conductors in $2\theta = \frac{2\theta}{360} Z$

Totally the angular degrees accomodating demagnetising ampere turns $= 4\theta$

Total demagnetising conductors $= \frac{4\theta}{360} Z$.

Total number of turns in 4φ angles $= \frac{2\theta Z I}{360}$

This gives the total demagnetising ampere turns for one pair of poles.

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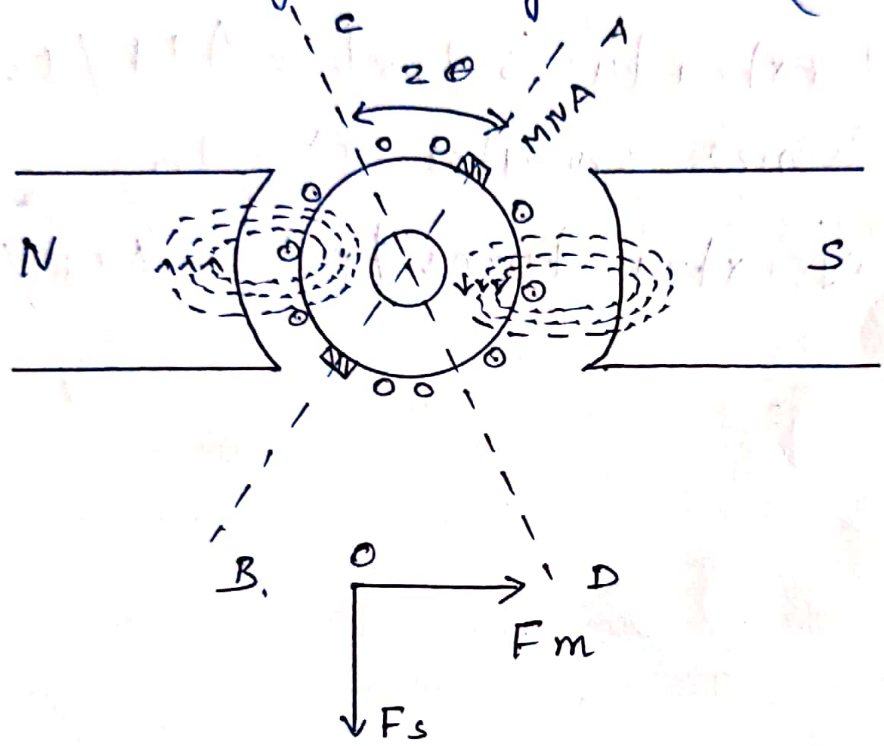
Demagnetising ampere-turns/pole $= \frac{2\theta}{360} \frac{ZI}{2}$

Demagnetising ampere-turns/pole $= \frac{\theta}{360} ZI$

$$\boxed{AT_d = \frac{\theta}{360} ZI}$$

Cross Magnetizing MMF (AT_c)



$Z \rightarrow$ Total number of conductors.

Total armature conductor per pole $= Z/P$

Demagnetising conductors per pole $= \frac{2\theta}{360} Z$

$\therefore$ Cross magnetising conductors per pole

$$= \frac{Z}{P} - \frac{2\theta}{360} Z$$

$$= Z\left[\frac{1}{P} - \frac{2\theta}{360}\right]$$

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∴ Cross magnetising ampere turns/pole

$$= \frac{ZI}{2}\left\{\frac{1}{P} - \frac{2\theta}{360}\right\}$$

For a shunt generator, extra turns per pole to be provided.

No. of extra turns / pole = AT_d / I_{sh}

For series generator $I_f = I_a$

No. of extra turns / pole = AT_d / I_a.

2/ed/pu

Example: A 250 kW, 400 V, 6 pole dc generator has 720 lap wound conductors. It is given a brush lead of 2.5 angular degrees (mech) from the geometric neutral. calculate the cross and demagnetizing turns per pole. Neglect the shunt field current.

solution :-

Armature current $I_a = \frac{P}{V} = \frac{25 \times 10^3}{400}$

I_a (or) $I_L = 625\ A$

Number of parallel path $= 6$

Conductor current $I_c = \frac{I_a}{A} = \frac{625}{6}$

$I_c = 104.2\ A$

Number of Commutator segments $= \frac{Z}{A} = \frac{720}{6} = 120$

Total armature ampere turns $AT_a = \frac{1}{2} \frac{Z \times I_c}{A}$

$= \frac{1}{2} \times \left(\frac{120 \times 104.2}{6}\right)$

$= 6252\ AT/pole.$

$\theta_m = 2.5°$ $\quad \theta = 2.5 (6/2) = 7.5°$ elect.

Demagnetising amp turn / pole $= 2 I \frac{\theta_m}{360}$

$= 720 \times 104.2 \times \frac{2.5}{360}$

$= 521\ AT/pole.$

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Cross - magnetizing amp - turn/pole $= ZI\left[\frac{1}{2P} - \frac{\theta}{360}\right]$

$= 720 \times 104.2\left[\frac{1}{2\times 6} - \frac{2.5}{360}\right]$

$= 75024\left[\frac{1}{12} - \frac{2.5}{360}\right]$

$= 75024 \times 0.0763$

$= 5731$ AT/pole.

Example: A 240 kW, 500V, 6-pole, lap wound DC generator has 63 slots with 10 conductors per slot. the brushes are advanced through 4 mechanical degrees. Ignoring shunt field current, find, (i) Demagnetising ampere turns/pole (ii) cross-magnetising ampere turns/pole.

Given:-

Power rating $= 240$ kW

Terminal voltage $= 500$V

Number of poles $P = 6$

For lap connected $A = P = 6$

Number of conductors $Z = 63 \times 10 = 630$

Mechanical degree $\theta_m = 4°$.

Solution :-

Load current $I_L = \frac{P_o}{V} = \frac{240\times10^3}{500} = 480A$

Here, neglect shunt field current $I_a = I_L = 480A$

Current in each conductor $I = \frac{I_a}{A} = \frac{480}{6} = 80A$

Demagnetising ampere turns/pole $= 2I\frac{\theta_m}{360}$

$= 630\times80\times\frac{4}{360}$

$\boxed{AT_d/\text{pole} = 560 \; AT/\text{pole}}$

Cross magnetising ampere turns/pole $= 2I\left[\frac{1}{2P} - \frac{\theta_m}{360}\right]$

$= 630\times80\left(\frac{1}{2\times6} - \frac{4}{360}\right)$

$\boxed{AT_c/\text{pole} = 3640 \; AT/\text{pole}}$

Example The brushes of a 400kW, 500V, 6-pole DC generator are given a lead of 12° electrical. calculate (1) the demagnetising ampere turns (2) the cross magnetising ampere-turns and (3) series turns required to balance the demagnetising component. The machine has 1000 conductors and the leakage co-efficient is 1.4.

Given Data:

Power rating = 400kW, Terminal voltage $V = 500V$,
Number of poles $P = 6$, $\theta_e = 12^\circ$, $Z = 1000$
Leakage coefficient = 1.4.

solution,

Armature current $I_a = \frac{P}{V} = \frac{400 \times 10^3}{500} = 800\,A$

For Lap connection identification
↳ Given current rating is higher.

Current in each conductor $I = \frac{I_a}{A} = \frac{800}{6} = 133.33\,A$

$$\theta_m = \frac{2\theta_e}{P} = \frac{2 \times 12}{6} = 4^\circ$$

Demagnetizing ampere turns/pole

$$= \frac{ZI\theta_m}{360} = 1000 \times 133.33 \times \frac{4}{360}$$

$$\boxed{AT_d/\text{pole} = 1481.4 \text{ AT/pole}}$$

Cross-magnetising ampere-turns/pole

$$= ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$$

$$= 1000 \times 133.33\left(\frac{1}{2 \times 6} - \frac{4}{360}\right)$$

$$\boxed{AT_c/\text{pole} = 9629.38 \text{ AT/pole}}$$

Number of extra series turns/pole $= \frac{AT_d}{I}\lambda$

$$= \frac{1481.4}{133.33} \times 1.4 = 15.55$$

Hence, $\boxed{\text{series turns required} = 15.55}$

Commutation:-

→ In DC generator the emf induced as well as the current flowing in the internal circuit is alternating.

→ To make the current flow unidirectional in the external circuit, split rings (or) Commutators are used.

→ The process by which the current in the short circuited coil of the armature gets reversed along MNA is called Commutation.

Mechanical causes of commutation:-

— Any mechanical imbalance of Commutator segments may occur projection of bars.

— Vibration on brush holder may also aggravate the chances of sparkover across the brushes.

Electrical causes of Commutation.

-An increase in the voltage across the Commutating bars may exceed the permitted limits. This may cause a high sparking current.

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- Any increase in the voltage across the commutator segments.
- Poor reactance voltage.

T_c – Time taken for the commutator to move a distance equal to the circumferential thickness of the brush minus the thickness of one insulating layer of Mica.

$$T_c = \frac{W_b - W_m}{V_c} \text{ seconds.}$$

Total change in current $= 2I$

$\therefore$ Rate of change in current $= \frac{2I}{T_c}$

$\therefore$ Induced emf $e_{react} = L \cdot \frac{dI}{dt}$

$$= L \cdot \frac{2I}{T_c}$$

For Linear

$$e_{react} = 1.11 \, L \cdot \frac{2I}{T_c}$$

For sinusoidal.

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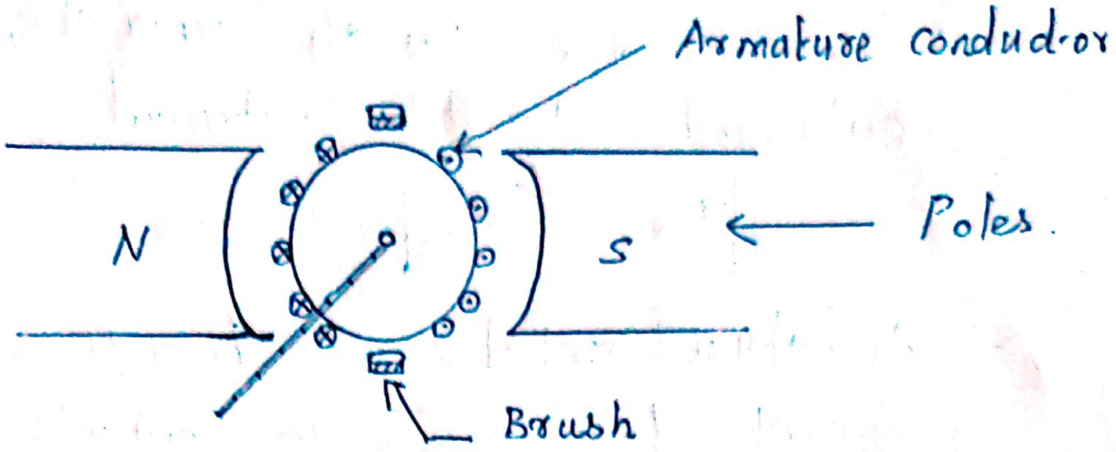
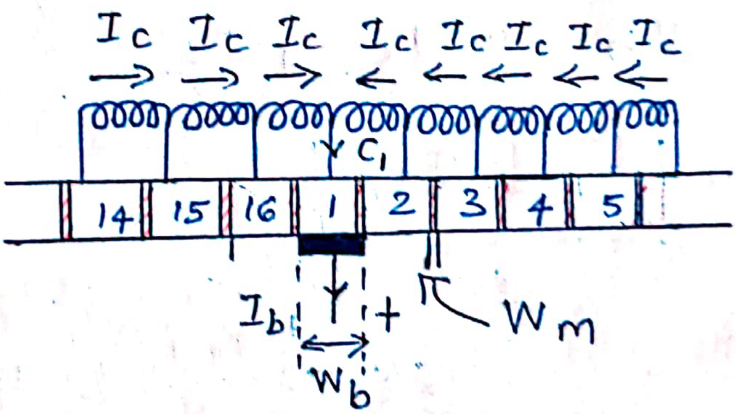
Process of Commutation:-



The armature coil ends are connected to the commutation segments and in turn brushes collect the current from internal circuit to the external circuit.



I_c – Coil current in amperes

W_b – width of the brush in cm.

W_m – width of the mica layer in cm.

I_b – brush current in amps.

There are 16 commutator segments in the machine,

$$W_c = W_b - W_m$$

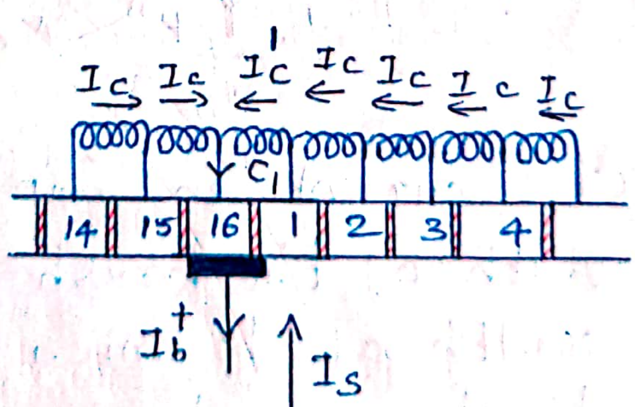
- C_1 is connected to segment 16 and 1.
- Brush is +ve with current flowing outwards to the external circuits.

$$\boxed{I_b = 2I_c}$$

- Armature rotates in clockwise direction
- segment 16 comes in contact with brush
- 16 comes in contact with brush C_1 partly enters into short circuited conditions.
- Collects the current I_c from 16 and 1.

$$I_b = 3 \cdot \frac{I_c}{2} + \frac{I_c}{2}$$

$$\boxed{I_b = 2I_c}$$



- The coil C_1 is almost at the end of Commutation period.

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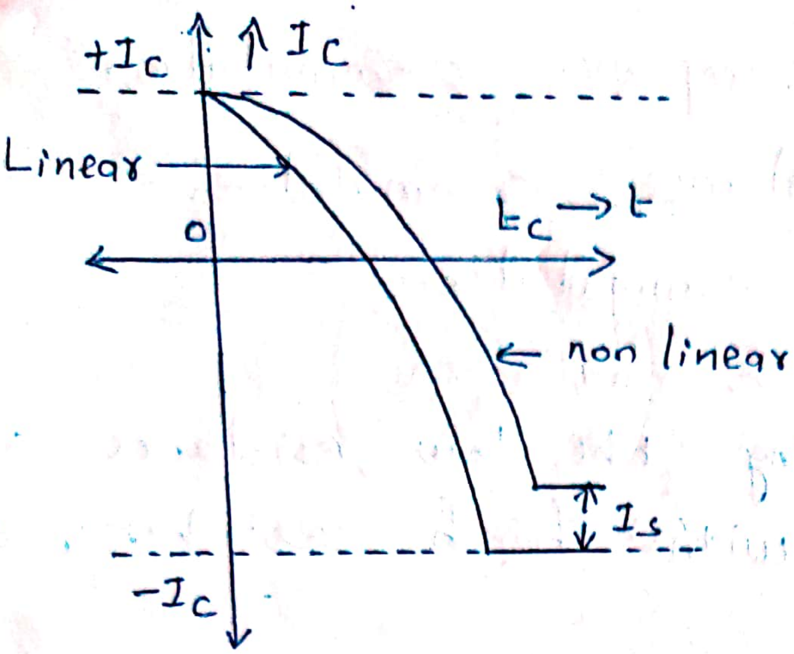

Ideal commutation:-

- The current should reverse from $+I_c$ to $-I_c$ exactly within the period of commutation.
- The change in current should take place in a linear fashion.

Delayed commutation:-

→ The current should reverse from $+I_c$ to $-I_c$ exactly within the period of commutation (t_c).

→ The change in current should take place in nonlinear fashion.

→ This is also called under commutation.

→ $|I_s| = |I_c| - |I_c'|$

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Method to improve commutation.

(i) Resistance commutation

(ii) EMF commutation.

Resistance commutation.

- Replacing the low resistance copper brushes with high resistance carbon brushes.

Advantages of carbon brushes:-

- act as self lubricating brush.
- Polishes the commutator segments.

Disadvantages of carbon brushes.

- Contact resistance is high, voltage drop is considerable.
- Larger commutations are required.

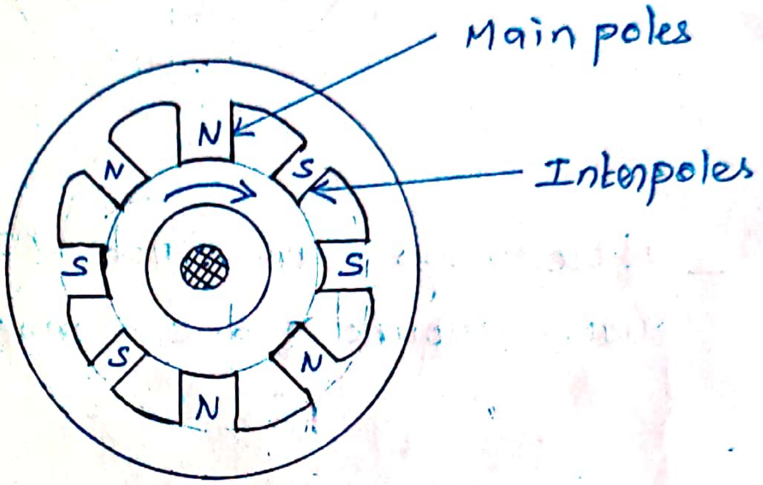
EMF commutation

- Reactance value is the main cause for sparking.
- Reactance voltage should be neutralised for ideal commutation
- This can be achieved by using interpoles.

$$AT_i = AT_a(\text{peak}) + \frac{B_i}{\mu_0} l_{gi}$$

Interpoles:-

- small poles, fixed to yoke and spaced in between the main poles.
- Wound with heavy gauge copper with turns.



- Interpoles are neutralize the cross magnetization effect of armature reaction.

Equilizer Connections.

- Form of thick copper rings at the back of the armature so as to reduce the variations in current flowing through the brushes.
- To avoid uneven distribution of current, flux. equilizer is provided.
- Number of connections that can be made to each equilizer ring has to be limited to number of pair of poles.

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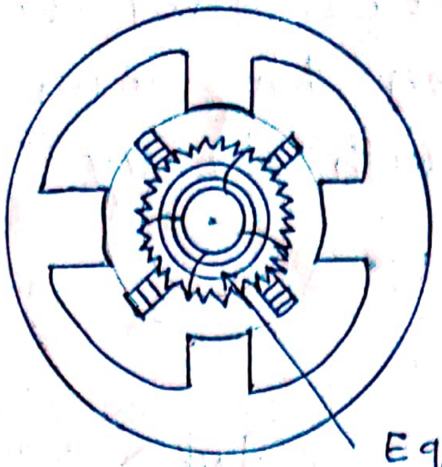

- These rings are not necessary in a wave wound d.c machines.

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ple: A 440V, 4 pole, 25 kw, dc generator has a wave-connected armature winding with 846 conductors. The mean flux density in the air gap under the interpoles is 0.5 wb/m² on full load and the radial gap length is 0.3 cm. calculate the number of turns required on each interpole.

Given data

$V = 440V$

Poles, $P = 4$

Power = 25 kw

wave connected.

Conductors, $Z = 846$

$B = 0.5\ wb/m^2$

$l_g = 0.3\ cm$.

Solution:-

$$AT_i = AT_a\ (\text{peak}) + \frac{B_i}{\mu_0} l_{gi}$$

$$= \frac{I_a Z}{2AP} + \frac{B_i}{\mu_0} l_{gi}$$

Assume $I_a = I_L = \frac{P}{V} = \frac{25\times10^3}{440}$

$= 56.82\ A$

$$AT_i = \frac{56.82 \times 846}{2\times2\times4} + \frac{0.5\times0.3\times10^{-2}}{4\pi\times10^{-7}}$$

$AT_i = 4198$

$$N_i = \frac{AT_i}{I_a}$$

$$= \frac{4198}{56.82} = 73.88$$

$\therefore \boxed{N_i = 74}$ number.

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Example A 100 KW, 250-V, 400A, long shunt compound generator has an armature resistance (including brushes) of 0.025 Ω, a series field resistance of 0.005 Ω, There are 1000 shunt field turns per pole and 3 series field turns per phase. The series field is connected in such a fashion that positive armature current produces direct current mmf which adds to that of the shunt field.

Compute the terminal voltage at rated terminal current when the shunt field current of 4.7 A and the speed is 1150 r/min. Neglect the effects of armature reaction.

Solution :-

For a long shunt machine

$$I_s = I_a = I_L + I_f$$

$$= 400 + 4.7 = 405 \text{ A}$$

$$\text{Gross mmf} = I_f + \left(\frac{N_s}{N_f}\right) I_s$$

$$= 4.7 + \left(\frac{3}{1000}\right) 405$$

$= 5.9$ equivalent shunt field amperes.

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By Examining the $I_a = 0$

$$E_g = 274\,V$$

$$E_a = \left(\frac{n}{n_0}\right) E_{a0}$$

$$= \frac{1150}{1200} \times 274 = 263\,V$$

Then $V_t = E_a - I_a(R_a + R_s)$

$$= 263 - 405(0.025 + 0.005)$$

$$= 251\,V$$

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UNIT III

DC MOTORS.

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UNIT - V

DC MOTORS

Introduction:

→ while a DC generator converts mechanical energy in the form of rotation of the conductor into electrical energy, a motor does the opposite.

→ The input to a DC motor is electrical and the output is mechanical rotation or torque.

→ The fundamental principles and construction of the DC motors are identical with DC generators which have the same type of excitation.

→ A DC machine that runs as a motor will also operate as a generator.

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Applications of DC generator.

1. Battery charging.
2. Boosters for adding a voltage to the transmission line.

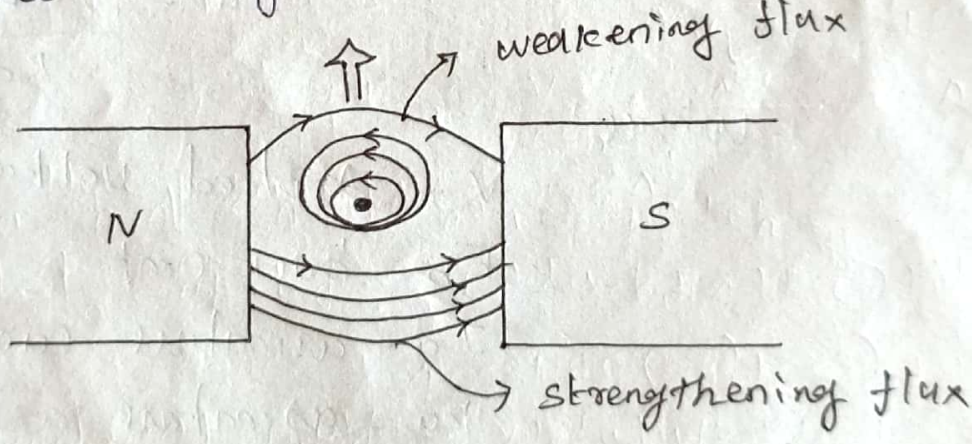
DC Motors.

DC motors converts electrical energy into mechanical energy.

Electrical Energy $\xrightarrow{(I/P)}$ DC Motor $\xrightarrow{(O/P)}$ Mechanical Energy

Principle:

Whenever a current carrying conductor is placed in a magnetic field, it experiences a force tending to move it.



The magnitude of the force experienced by the conductor in a motor is given by,

$$F = BIl \text{ Newton}$$

where B = Magnetic field intensity in wb/m^2

I = current in Amps

l = length of the Conductor in metres.

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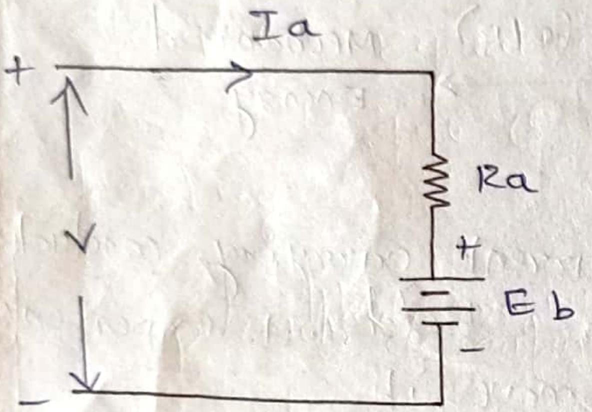
Back emf:

Even when the machine is working as a motor, voltages are induced in the conductors. This emf is called the back emf or counter emf. According to Lenz's law, the direction of the back emf opposes the supply voltage.

The back emf is $E_b = \frac{\phi Z N}{60} \times \frac{P}{A}$ volts.



The voltage equation of this DC motor is

$$V = E_b + I_a \cdot R_a \text{ volts.}$$

From this equation,

$$I_a = \frac{V - E_b}{R_a} \text{ Amps.}$$

where

V - Applied voltage

Eb - back emf

Ia - armature current

Ra - armature resistance

Importance of Back EMF:

1. When the dc motor is operating on no load condition, small torque is required to overcome the friction and windage losses. Therefore the back emf is nearly equal to input voltage and armature current is small i.e. Ia is Low.

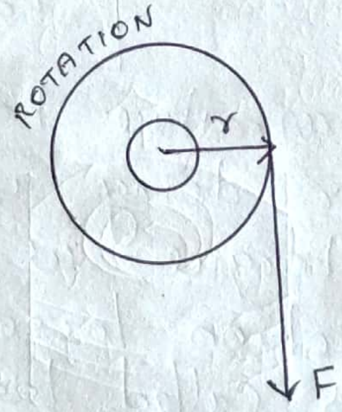
2. when the DC motor is operating on load, armature slows down and motor back emf E_b also decreases. corresponding armature current I_a increases.

3. when the load on DC motor is decreased, motor speed increases, the back emf E_b also increases causing armature current to decrease.

Torque Equation.

Torque is nothing but turning or twisting force about an axis.

Torque = Force x radius N-m



The angular velocity $\omega = \frac{2\pi N}{60}$ rad/sec.

workdone per revolution = F x distance moved

$= F \times 2\pi r$ joules

power developed $P = \frac{\text{work done}}{\text{time}}$

$= \frac{F \times 2\pi r}{60/N}$

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$$P = (F \times r) \frac{2\pi N}{60}$$

$P = T\omega$ watts.

where T = torque in N-m

ω = angular speed in rad/sec.

The gross mechanical power developed in the armature is $E_b I_a$. Then

power in armature = Armature torque $\times \omega$

$$E_b I_a = T_a \times \frac{2\pi N}{60} \; ; \; E_b = \frac{\phi P N Z}{60 A}$$

$$\frac{\phi P N Z}{60 A} I_a = T_a \times \frac{2\pi N}{60} \; ;$$

$$T_a = \frac{\phi I_a}{2\pi} \frac{P Z}{A}$$

$$\boxed{T_a = 0.159 \phi I_a \frac{P Z}{A} \text{ N-m}}$$

The full armature torque is not available for doing useful work. Some amount of torque is used for supplying iron and friction losses in the motor. This torque is called lost torque. The remaining torque is available in the shaft. It is used for doing useful work.

The armature torque is the sum of the lost torque and shaft torque.

$$\therefore T_a = T_f + T_{sh}$$

The output power of the motor is

$$P_{out} = T_{sh} \times 2\pi N \text{ watts.}$$

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Power Relationship of DC Motor.

The voltage equation is $V = E_b + I_a R_a$ —— ①

Multiplying each term of the voltage equation by I_a, we get

$$V I_a = E_b I_a + I_a^2 \cdot R_a \quad \text{—— ②}$$

This equation is known as power equation of a DC motor.

$VI_a \rightarrow$ Electric power supplied to armature.

$E_b I_a \rightarrow$ Power developed by the motor armature.

$I_a^2 R_a \rightarrow$ Power loss in the armature.

Mechanical power developed $P_m = E_b I_a$ —— ③

$$= V I_a - I_a^2 R_a \quad \text{—— ④}$$

Differentiating both sides with respect to armature current I_a we have

$$\frac{dP_m}{dI_a} = V - 2 I_a R_a \quad \text{—— ⑤}$$

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For maximum mechanical power, $\frac{dP_m}{dI_a}$ is zero,

$$\text{or} \quad V - 2I_aR_a = 0$$

$$I_aR_a = \frac{V}{2}$$

$$V = E_b + I_aR_a$$

$$V = E_b + \frac{V}{2}$$

$$\boxed{E_b = \frac{V}{2}} \quad \text{—(6)}$$

Therefore the power developed in armature is maximum when the back emf is equal to half of the input voltage.

Disadvantages:

This is not in practice, because

→ The motor armature (I_a) current is very large.

→ Half of the input power is wasted in the armature.

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For DC shunt motor

ϕ = constant

$\therefore \boxed{T \propto I_a}$

For series motor

$\phi \propto I_a$

$\therefore \boxed{T \propto I_a^2}$

Applications of DC motor.

Shunt motor → Centrifugal pumps, light machine tools, wood working machines, lathe etc.

Series motor → Cranes, hoists, fans, blowers, conveyers, lifts etc.

Compound motor → Intermittent shears, punching machies etc.

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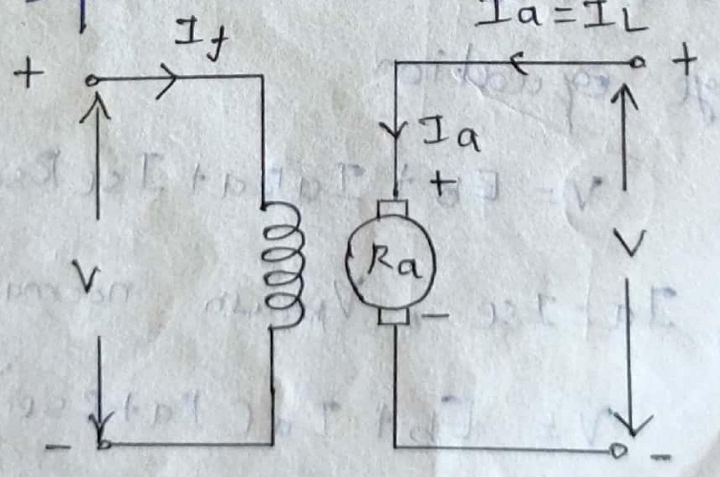
Types of DC Motors:

The classification of DC motor is similar to that of DC generators. They are,

1. seperately excited DC motor
2. self excited DC motor
   a. series motor
   b. Shunt motor
   c. compound motor
      1. Long shunt
      2. short shunt

1. Seperately excited DC motor :-

Here the field winding and armature are seperated.



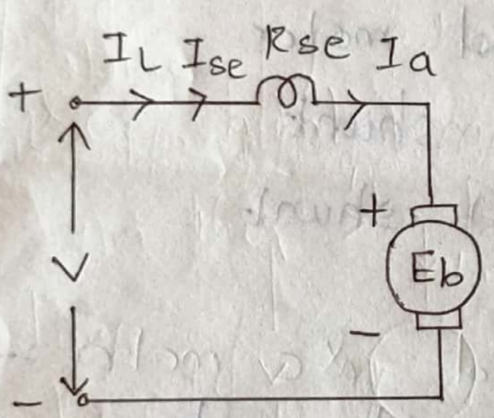
From the above diagram

Armature Current I_a = Line current I_L

Back emf $E_b = V - I_a R_a - V_{brush}$

DC series motor :-

DC series motor means, the field winding is connected in series with armature.



→ Field winding should have less number of turns of

→ Thick wire.

$$I_a = I_{se} = I_L$$

voltage equation

$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

$I_a = I_{se}$, V_{brush} normally neglected

$$V = E_b + I_a(R_a + R_{se})$$

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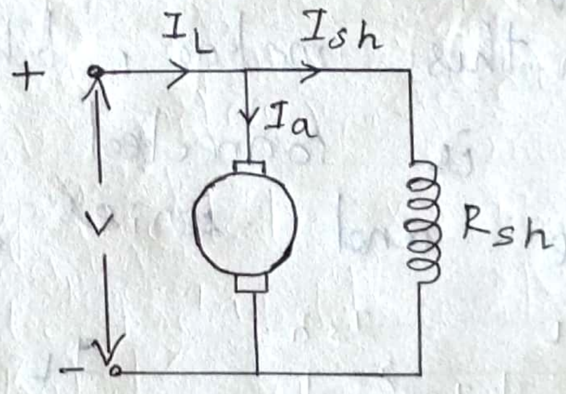
In a DC series motor, full armature current flows through the series field winding,

$$\therefore \boxed{\phi \propto I_{se} \propto I_a}$$

DC shunt motor:

→ In a DC shunt motor, the field winding is connected across the armature.



→ I_L is the line current drawn by the supply. $I_L = I_a + I_{sh}$

$$I_{sh} = \frac{V}{R_{sh}}$$

Voltage equation of a DC shunt motor is given by, $V = E_b + I_a R_a + V_{brush}$

→ In shunt motor, flux produced by field winding is proportional to the field current

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ie $\boxed{\phi \propto I_{sh}}$

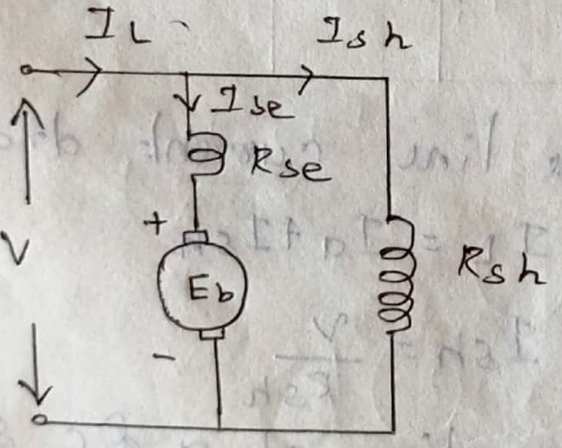
→ Therefore Dc shunt motor is also called a constant flux motor or constant speed motor

Dc Compound Motor:

A Dc compound motor consists of both series and shunt field windings.

a. Long shunt compound motor.

In this motor, the shunt field winding is connected across both armature and series field winding.



$I_L = I_{se} + I_{sh}$

$I_{se} = I_a$

$\therefore I_L = I_a + I_{sh}$

$I_{sh} = \dfrac{V}{R_{sh}}$

Voltage equation $V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$

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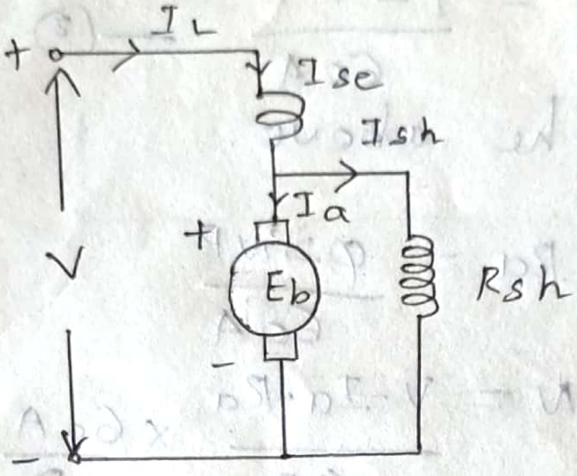
Short shunt Compound motor.

In this motor, the shunt field winding is across the armature and series field windings is connected in series with this combination.



$I_L = I_{se}, I_L = I_a + I_{sh}$

$\therefore I_L = I_{se} = I_a + I_{sh}$

$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$

$I_{se} = I_L$, voltage drop across shunt field winding $= V - I_L R_{se}$

$$\boxed{I_{sh} = \frac{V - I_L R_{se}}{R_{sh}}}$$

Cumulative compound motor.

The two field winding fluxes aid each other.

Differential Compound motor:-

The two field winding fluxes oppose each other.

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4. Speed and Torque equation:-

The speed equation is obtained as follows:

$$E_b = V - I_a \cdot R_a \quad \text{—} ①$$

$$E_g = \frac{\varphi Z N P}{60 A} \quad \text{—} ②$$

equate the above.

$$V - I_a \cdot R_a = \frac{\varphi Z N P}{60 A}$$

$$N = \frac{V - I_a \cdot R_a}{\varphi Z} \times \frac{60 A}{P}$$

Z, A & P are constant

$$N = \frac{k\,(V - I_a \cdot R_a)}{\varphi} \quad \text{—} ③$$

k is constant

speed eqn becomes

$$N \propto \frac{V - I_a \cdot R_a}{\varphi}$$

$$N \propto \frac{E_b}{\varphi} \quad \text{—} ④$$

The torque eqn of DC motor is given by,

$$T \propto \varphi I_a$$

$$\varphi \propto I_f$$

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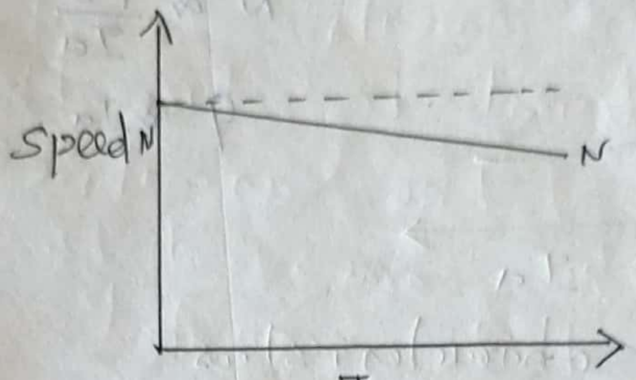
Characteristics of DC motors.

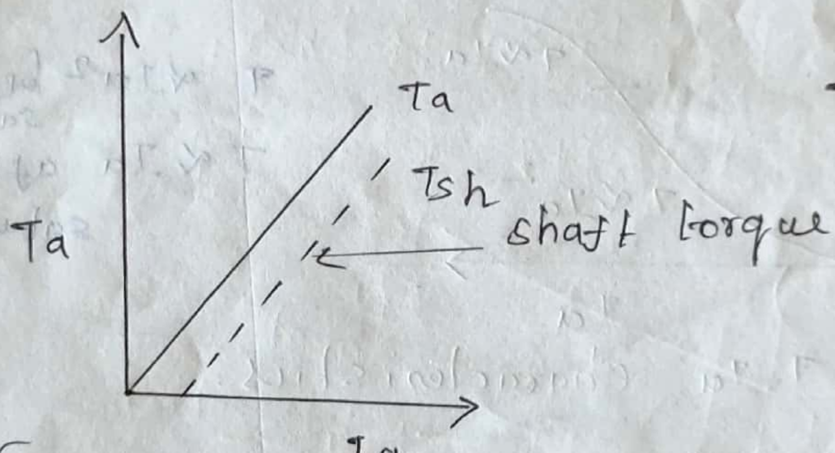
1. shunt motor characteristics.



→ speed $N = \frac{k(V - I_a R_a)}{\varphi}$

→ DC shunt motor acts as a constant speed motor.

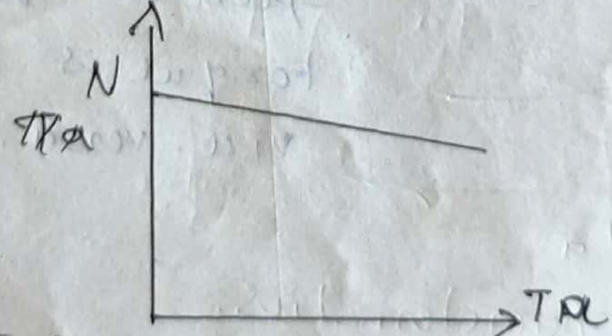
(a) N – Ia characteristics :



$T \propto \varphi I_a$

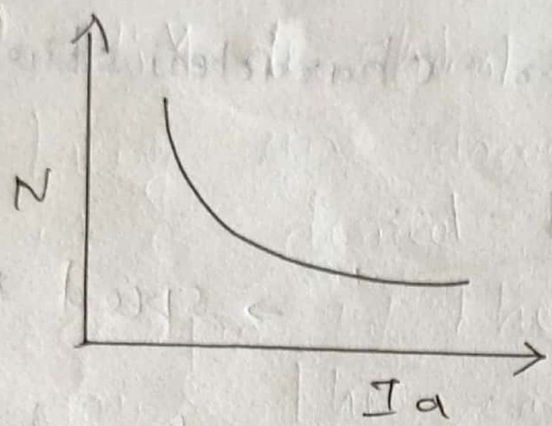
$T \propto I_a$

(b) Ta – Ia characteristics.



(c) N – Ta characteristics.

Dc series motor characteristics:

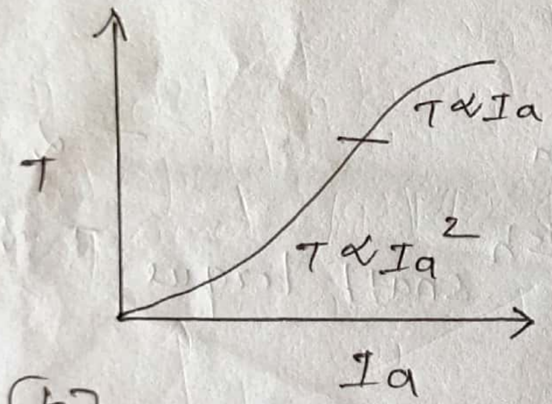


$N \propto \frac{E_b}{I_a}$

$\varphi \propto I_a$.

(a) N-Ia characteristics.

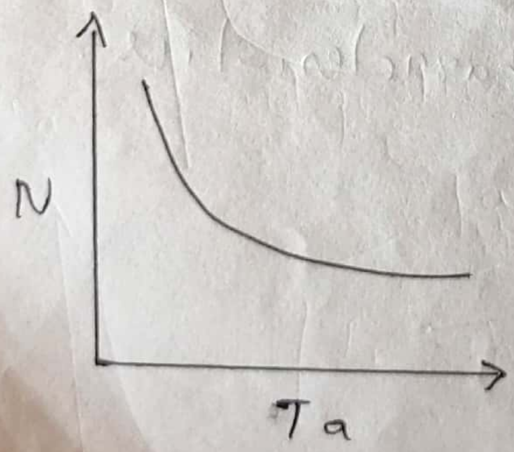
→ Dc motor should never be started without some load.



$T \propto I_a^2$ before saturation

$T \propto I_a$ after saturation.

(b) T-Ia characteristics.

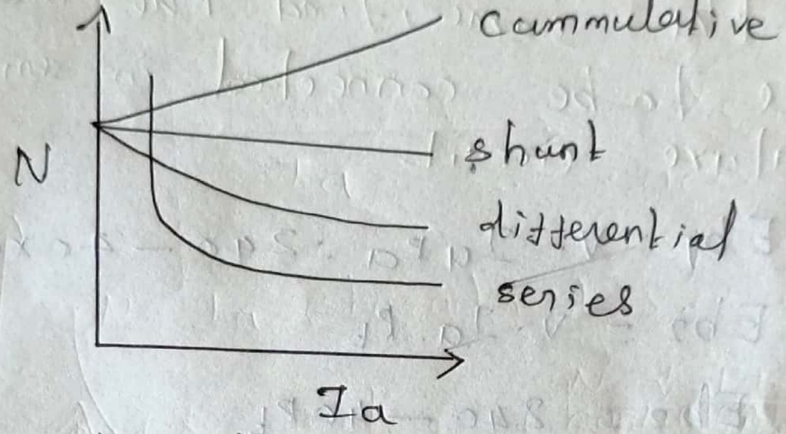


→ Dc series motor speed is high, the torque is low and vice-versa.

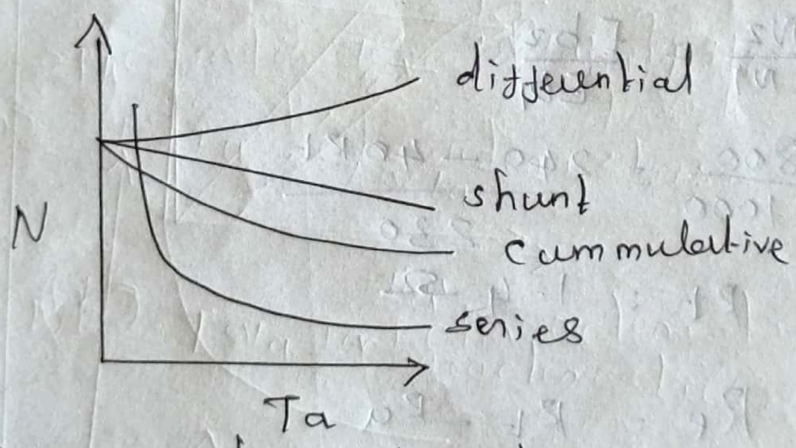
(c) N-Ta characteristics.

3. Compound motor characteristics.

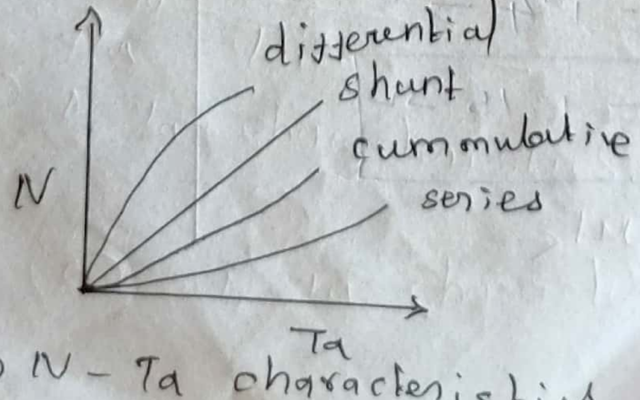
The characteristics of compound motor will depend on whether the series and shunt field windings are assisting each other or opposing each other.



(a) N- Ia - characteristics.



(b) N-Ta characteristics.



(c) N - Ta characteristics.

A 240 V dc shunt motor has an armature resistance of 0.25 Ω and runs at 1000 rpm taking an armature current of 40 A. It is desired to reduce the speed to 800 rpm. If the armature current remains the same, find the additional resistance to be connected in series with the armature circuit.

Soln:- $E_{b1} = V - I_a R_a = 240 - 40 \times 0.25 = 230V$

$$E_{b2} = V - I_a . R_t$$

$$E_{b2} = 240 - 40 R_t$$

$$\phi_1 = \phi_2$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

$$\frac{800}{1000} = \frac{240 - 40 R_t}{230}$$

$$R_t = 1.4 \Omega$$

$$R_e = R_t - R_a$$

$$= 1.4 - 0.25$$

$$\boxed{R_e = 1.15 \Omega}$$

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Ex:1 A DC motor connected to a 460V supply has an armature resistance of 0.15 Ω calculate (a) the value of back emf when the armature ct is 120 A. (b) the value of armature ct when the back emf is 447V.

Gn:-

$V = 460V$

$R_a = 0.15\,\Omega$

$I_a = 120\,A$

(a) $E_b = V - I_a \cdot R_a$

$E_b = 460 - (120 \times 0.15)$

$\boxed{E_b = 442V}$ when $I_a = 120\,A$.

(b) $I_a \cdot R_a = V - E_b$

$$I_a = \frac{V - E_b}{R_a}$$

$$= \frac{460 - 447}{0.15} = \frac{13}{0.15}$$

$\boxed{I_a = 86.67\,A.}$

Ex2. A 4 pole 250V series motor has a wave connected armature with 1254 conductors. The flux per pole is 22 mwb. The motor takes an armature ct of 50 A. Armature and field resistances are 0.2 Ω and 0.2 Ω respectively. Calculate its speed.

Gn: $P = 4$, $V = 250V$, $Z = 1254$, $\phi = 22$ mwb

$I_a = 50A$, $R_a = 0.2\,\Omega$, $R_{se} = 0.2\,\Omega$, $A = 2$

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soln:-

$$E_b = V - I_a(R_a + R_{se})$$

$$= 250 - 50(0.2 + 0.2)$$

$$= 250 - 20 = 230V$$

$$E_b = \frac{\phi P N Z}{60 A}$$

$$N = \frac{E_b \times 60 A}{\phi P Z}$$

$$= \frac{230 \times 60 \times 2}{22 \times 10^{-3} \times 4 \times 1254}$$

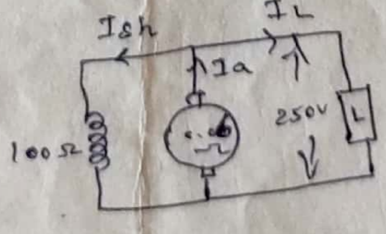
$$\boxed{\text{speed} = 250 \text{ rpm.}}$$

Ex: A 25kW, 250V, DC shunt generator has armature and field resistances of 0.06Ω and 100Ω respectively. Determine the total armature power developed when working
(1) as a generator delivering 25kW o/p
(2) as a motor taking 25kW i/p.

Gn: $V = 250V$, $P = 25kW$, $R_a = 0.06\Omega$, $R_{sh} = 100\Omega$

soln:-

$$I_L = \frac{25 \times 10^3}{250}$$
$$= 100 A$$



$P_o = 25kW$

$$I_{sh} = \frac{V}{R_{sh}} = 2.5 A$$

$$I_a = I_L + I_{sh} = 102.5 A$$

$$E_g = V + I_a . R_a = 250 + 102.5 \times 0.06$$

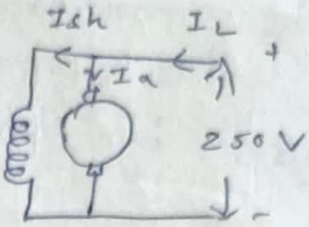
$$E_g = 256.15 V$$

$$P_a = E_g I_a$$

$$= 256.15 \times 102.5$$

$$P_a = 26255.375\ W$$

2) As a motor.



$$I_L = \frac{25 \times 10^3}{250}$$

$$= 100\ A$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{100} = 2.5\ A$$

$$I_a = I_L - I_{sh} = 100 - 2.5 = 97.5\ A$$

$$E_b = V - I_a \cdot R_a$$

$$= 250 - 97.5 \times 0.06$$

$$E_b = 244.15\ V$$

power developed in the armature $= E_b \cdot I_a$

$$= 244.15 \times 97.5$$

$$\boxed{P_a = 23804.625\ W}$$

Ex4: A 4 pole dc motor takes an armature of 50 A. The armature has lap connected 480 conductors. The flux per pole is 20 mwb. calculate the gross torque developed by the motor.

Soln:- $T_a = 0.159\,\phi I_a \frac{PZ}{A}$

$$= 0.159 \times 20 \times 10^{-3} \times 50 \times \frac{4 \times 480}{4}$$

$$\boxed{T_a = 76.32\ N\text{-}m}$$

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Ex.5: A 200V, 2000 rpm, 10 A, seperately excited dc motor has an armature resistance of 2 Ω. Rated dc vge is applied to both the armature and field wdg of the motor. If the armature draws 5A from the source, calculate the torque developed by the motor.

Gn: $V = 200V$ $N_1 = 2000$ rpm
$R_a = 2\ \Omega$ $I_{a2} = 5A$
$I_{a1} = 10A$

soln:-

$E_{b2} = V - I_{a2} \cdot R_a = 200 - 5\times 2 = 190V$

$E_{b2} = V - I_{a1} R_a = 200 - 10\times 2 = 180V$

$N_2 = \frac{190}{180} \times 2000$

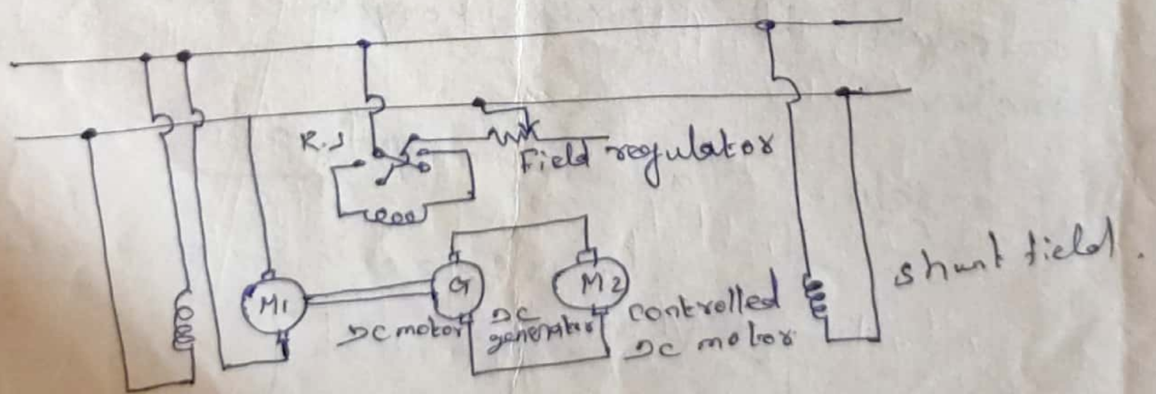
$N_2 = 2111$ rpm.

$E_b I_{a2} = T_a \omega_2 = T_a \cdot \frac{2\pi N_2}{60}$

$T_a = \frac{E_b I_{a2} \cdot 60}{2\pi N_2}$

$\boxed{T_a = 4.29\ N\text{-}m}$

ward - Leonard Control System.



EXAMPLE: A 500V DC shunt motor with constant field drives a load whose torque is proportional to the square of the speed. When running at 900rpm it takes an armature current of 45A. Find the speed at which the motor runs if a resistance of 8 Ω is connected in series with the armature. The armature resistance may be taken as 1 Ω. (NOV/DEC 2019)

Given Data

Voltage = 500V

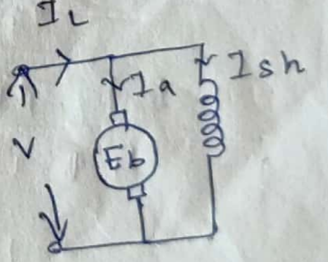
$N = 900$ rpm

$I_a = 45$ A

$R_{external} = 8\,\Omega$

$R_a = 1\,\Omega$

Solution



$E_b = V - I_{a1} R_a$

$E_b = 500 - 45 \times 1$

$= 500 - 45$

$\boxed{E_b = 455V}$

$E_b = V - I_{a2}(R_a + R_{ext})$

$455 = 500 - I_{a2}(1 + 8)$

$9 I_{a2} = 500 - 455$

$I_{a2} = 45/9 = 5$ A

$T \propto N^2$

$T \propto \phi I_{a_1}$

$T_1 \propto I_{a_1}^2$

$T_2 \propto I_{a_2}^2$

$$\left(\frac{I_{a_2}}{I_{a_1}}\right)^2 = \left(\frac{N_2}{N_1}\right)^2$$

$$N_2^2 = N_1^2 \times \left(\frac{I_{a_2}}{I_{a_1}}\right)^2$$

$$= 900^2 \times \left(\frac{5}{45}\right)^2$$

$$N_2^2 = 10000$$

$$\boxed{N_2 = 100 \text{ rpm}}$$

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Necessity of a starter:-

The voltage equation of a DC motor is $V = E_b + I_a \cdot R_a$

$$I_a = \frac{V - E_b}{R_a}$$

At the instant of starting $E_b = 0$.

$$\therefore I_a = \frac{V - 0}{R_a} = \frac{V}{R_a}$$

At starting the motor takes large amount of current. which is nearly 25 times the full load current.

But it is can't be allow in a motor for a short time period.

It causes damage the brushes and commutator and brush gear.

It limits the starting current due to a safe value.

Types of DC motor starters:-

1. Two point starter
2. Three point starter
3. Four point starter.

Two point starter:-

No Load Release coil (NLR)

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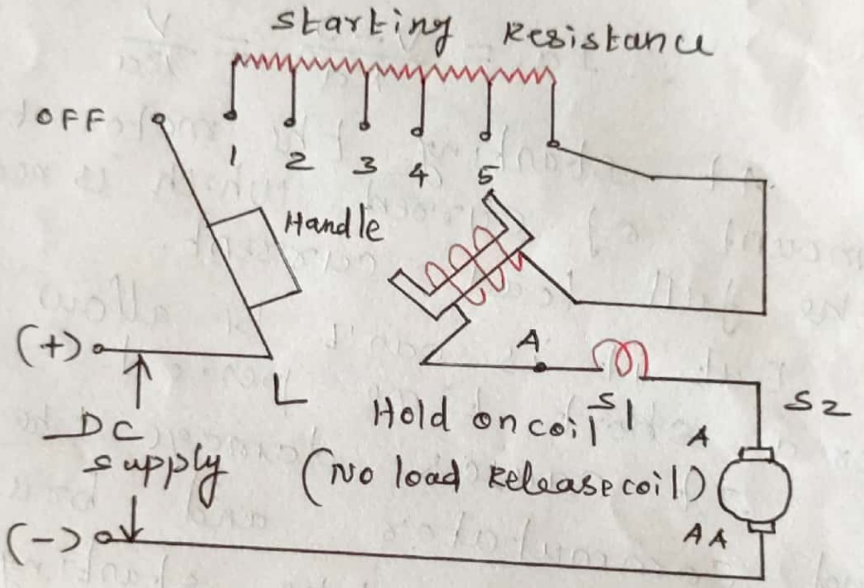
* The Load current flows through NLR, It gives necessary protection to the motor.

* when load current becomes zero, NLR release the handles and back to OFF position.



* Here starting resistance is connected in series with the armature.
* No load release coil is connected in series with armature.
* After closing the supply the handle is moved from OFF to stud 1. It gives full resistance so the current is reduced.
* In this way the flow of current and increased gradually.

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Three point starter.

* It consists starting resistance R1 to R6 is connected in series with armature.
* Handle can be moved over there resistances.
* NVR Coil is connected in series with field winding.
* OLR (over load release) is connected in series with armature and movable arm is placed near OLR.

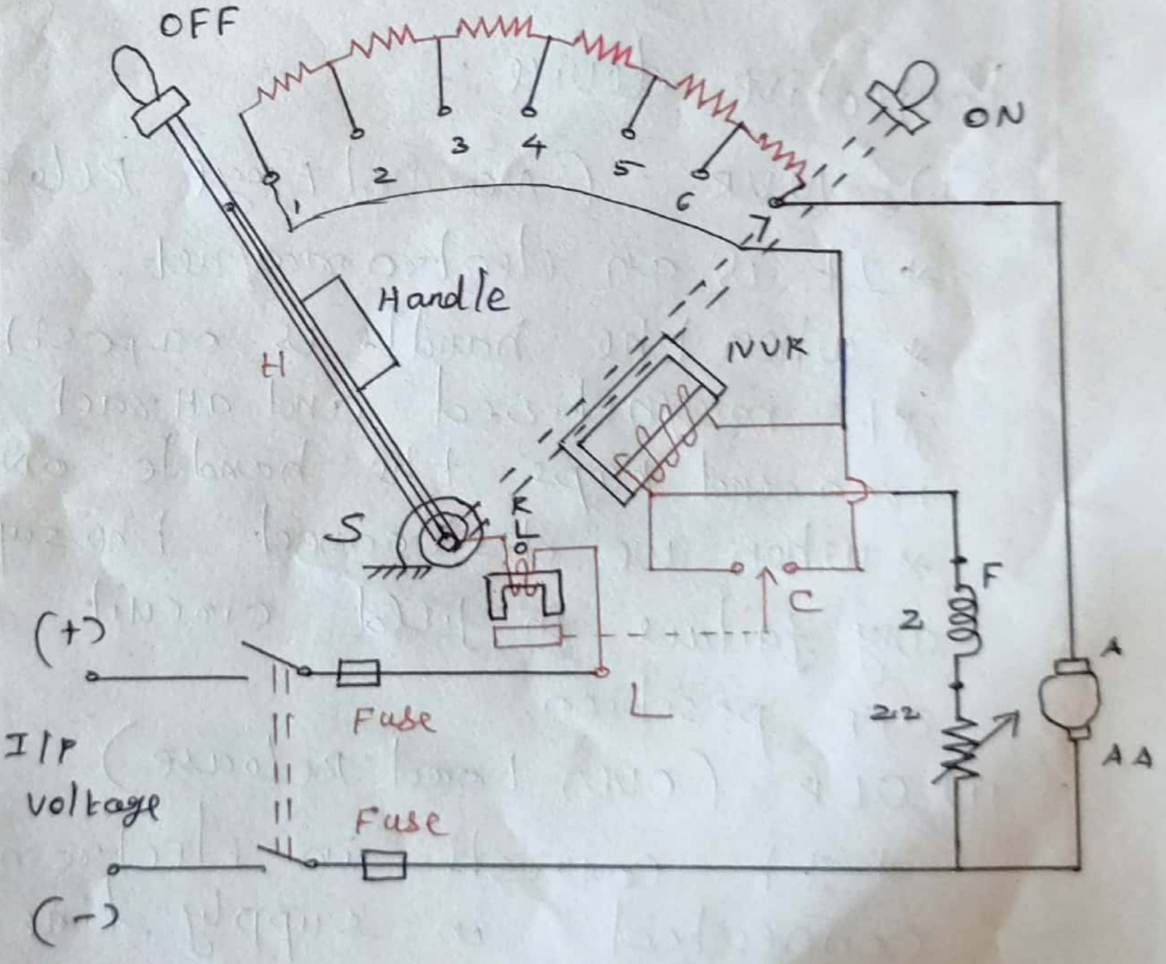

Fig: 3 point starter.

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operation :-

* when we give the supply voltage handle moved over the starting resistance.

* Now the handle is at stud No 1. so that it give full resistance.

* The starting current is reduced, the handle is further moved and the resistance cut out gradually.

* The motor develops the back emf when it gethers speed.

Protective device :-

(i) NVR : (No voltage Release)

*It is an electromagnet.

* when the handle is on position, it get magnetized and attract the soft iron and keeps the handle ON position.

* when we disconnect the supply or any failure in field circuit NVR goto OFF position.

(ii) OLR : (over Load Release)

* It consists an electromagnet connected in supply line.

* It lifted the arm to OFF position when the motor becomes overloaded.

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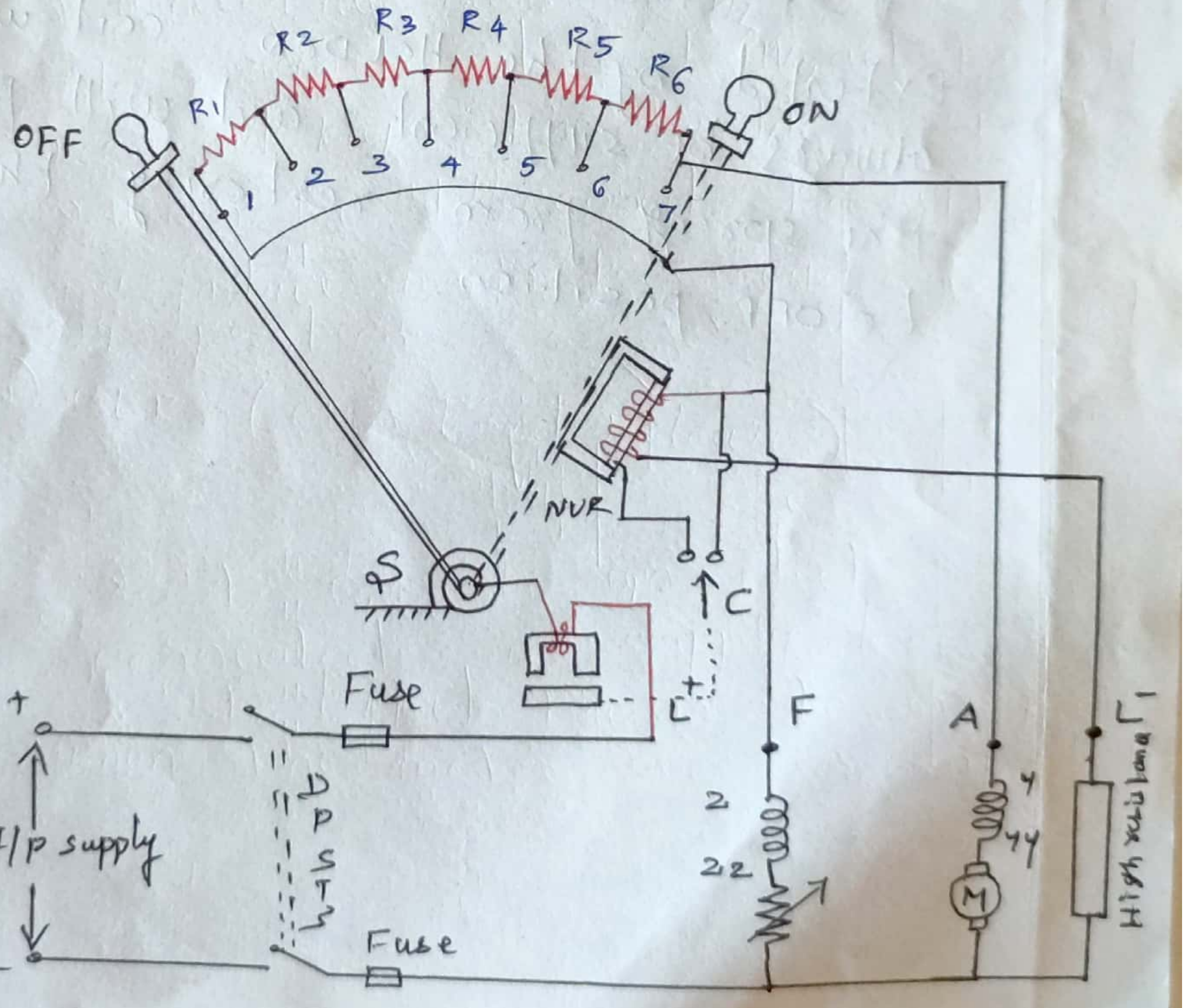
Demerits of 3 point starter.

$$N \propto \frac{1}{\phi}$$

* To achieve higher speed, the field current is to be reduced to very low.
* This low current losses through NVR. It is unable to create enough electromagnetic pull to overcome the spring tension.
* So that the arm is pulled back to OFF position.



Working :-

* In this starter the HOLD ON will has been taken out of the shunt field circuit.

* HOLD ON coil is directly connected across the supply line through protecting resistance (HR)

* So any change of current in shunt field circuit does not affect the current passing through the HOLD ON coil.

* It means the electromagnetic pull exterted by the HOLDON coil will always be sufficient and will prevent the spring from restoring the handle to OFF position.

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Determine developed torque and shaft torque of 220V, 4 pole series motor with 800 conductors wave connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mwb and its armature circuit resistance is 0.6 Ω

April/May 2018

Gn data

$V = 220V$, $P = 4$, $Z = 800$

$P_{out} = 8.2 kW$, $I_L = 45A$, $\phi = 25 mwb$.

$R_a = 0.6 \Omega$, $A = 2$

soln:-

$$T_a = 0.159 \phi \frac{I_a P Z}{A}$$

$$= 0.159 \times 25 \times 10^{-3} \times \frac{45 \times 4 \times 800}{2}$$

$$\boxed{T_a = 286.2 \ N\text{-}m}$$

$$T_{sh} = 9.55 \frac{P_{out}}{N}$$

$$E_b = V - I_a R_a = 220 - 45 \times 0.6 = 193V$$

$$E_b = \frac{P \phi Z N}{60 A}$$

$$N = \frac{E_b 60 A}{P \phi Z} = \frac{193 \times 60 \times 2}{4 \times 25 \times 10^{-3} \times 800}$$

$$= 289.5 \ rpm$$

$$T_{sh} = 9.55 \times \frac{8.2 \times 10^3}{289.5}$$

$$\boxed{T_{sh} = 270.5 \ N\text{-}m}$$

Scanned with CamScanner

A 440V DC shunt motor takes 4 A at no load. Its armature and field resistances are 0.4 Ω and 220 Ω respectively. Estimate the kW o/p and efficiency when the motor takes 60A on full load. (April/May 2018)

Given data

$V = 440\,V$, $I_0 = 4A$, $R_a = 0.4\,\Omega$, $R_{sh} = 220\,\Omega$

$I_L = 60A$.

soln:-

No load i/p power $= VI_0 = 440 \times 4 = 1760\,W$.

$$I_{sh} = \frac{V}{R_{sh}} = \frac{440}{220} = 2\,A$$

$$I_{a0} = I_0 - I_{sh} = 4 - 2 = 2\,A$$

$$P_{cu}\ loss = I_{a0}^2 \cdot R_a = 2^2 \times 0.4 = 1.6\,W$$

Constant loss $= 1760 - 1.6 = 1758.40\,W$

when the line ct is 60A,

$$I_a = I_L - I_{sh} = 60 - 2 = 58\,A$$

$$P_{cu} = I_a^2 R_a = 58^2 \times 0.4 = 1345.6\,W$$

Total loss at full load = P_{cu} + Constant loss

$= 1345.6 + 1758.4$

$= 3104\,W$

$$P_{in} = VI_L = 440 \times 60 = 26400\,W$$

$$P_{out} = P_{in} - \text{Total loss} = 26400 - 3104$$

$$P_{out} = 23.296\,kW$$

$$\boxed{\eta = \frac{o/p}{I/p} = \frac{23.296}{26.400} = 88.24\,\%}$$

(23)

A 220V, 22 A, 1000 rpm DC shunt motor has armature circuit resistance of 0.1 Ω and field resistance of 100 Ω. Calculate the value of additional resistance to be inserted in the armature circuit inorder to reduce the speed to 800 rpm. Assume the load torque to be (i) proportional to the speed and (ii) proportional to square of the speed.

(April/may 2018)

soln:-

(i) $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\varphi_1}{\varphi_2}$

Since the magnetic circuit is unsaturated, flux directly proportional to the shunt ct.

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}}$$

$$I_{sh} = \frac{220}{100} = 2.2 A$$

$$T_a \propto \varphi_1 I_{a1} \propto \varphi_2 I_{a2}$$

$I_L = 22 A$

$I_a = 22 - 2.2$

$I_a = 19.8 A$

If φ constant

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

$$E_{b1} = V - I_{a1} \cdot R_a = 220 - 19.8 \times 0.1 = 218.02 V$$

$$E_{b2} = V - I_{a2}(R_e + R_a)$$

$$= 220 - 19.8(R_e + 0.1)$$

$$= 220 - 19.8 R_e - 1.98$$

$$E_{b2} = 218.02 - 19.8 R_e$$

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$$\frac{218.02 - 19.8\,Re}{218.02} = \frac{800}{1000}$$

$$218020 - 19800\,Re = 174400$$

$$19800\,Re = 43620$$

$$\boxed{Re = 2.20\ \Omega}$$

(ii) $$\frac{218.02 - 19.8\,Re}{218.02} = \frac{800^2}{1000^2}$$

$$218.02 \times 10^6 - 19.8\,Re \times 10^6\ Re = 139532800$$

$$\boxed{Re = 3.96\ \Omega}$$

Scanned with CamScanner

A 220V d.c series motor has armature and field resistances of 0.15 Ω and 0.10 Ω respectively. It takes a current of 30 A from the supply while running at 1000 rpm. If an external resistance of 1 Ω is inserted in series with the motor. Calculate the new steady state armature current and the speed. Assume the load torque is proportional to the square of the speed i.e $T_L \propto n^2$. (April/May 2019)

Soln:-

Since the load torque remains constant in both cases, we have.

$$T_{e1} = T_{e2} = T_L$$

$$(or)\ k_t I_{a1}^2 = k_t I_{a2}^2$$

$$30^2 = I_{a2}^2$$

$$\boxed{I_{a2} = 30\ A}$$

$$E_{b1} = V - I_{a1}(r_{se} + r_a)$$

$$k_g I_{a1} n_1 = 220 - 30(0.1 + 0.15)$$

$$k_g\, 30 \times 1000 = 212.5\ V$$

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$$E_{b2} = V - I_{a2}(r_{se} + r_a + r_{ext})$$

$$K_g I_{a2} n_2 = 220 - 30(0.1 + 0.15 + 1)$$

$$K_g 30 n_2 = 182.5\,V$$

$$\frac{K_g \times 30 \times n_2}{K_g \times 30 \times 1000} = \frac{182.5}{212.5}$$

$$\therefore n_2 = \frac{182.5}{212.5} \times 1000$$

$$\text{or} \quad \boxed{n_2 = \frac{182500}{212.5} = 858.8\ \text{rpm}}$$

Scanned with CamScanner

Initially a d.c shunt motor having $R_a = 0.5\,\Omega$ and $R_f = 220\,\Omega$ is running at 1000 rpm drawing 20A from 220V supply. If the field resistance is increased by 5%, calculate the new steady state armature current and speed of the motor. Assume the load torque to be constant. (April/May 2019)

Soln:-

For initial operating point

$I_{L1} = 20\,A$, $r_a = 0.5\,\Omega$, $V = 220V$

$I_{sh1} = 220/220 = 1A$

$I_{a1} = 2 - 1 = 19A$

$E_{b1} = k_g I_{sh1} n_1 = k_g \times 1 \times 1000 = V - I_{a1} r_a$

$\frac{\phi_1 ANP}{60A}$ I_{sh1}

$= 220 - 19 \times 0.5$

$= 210.5\,V$

$R_{sh2} = 1.05 \times 220 = 231\,\Omega$

$I_{sh2} = \frac{220}{231} = 0.95\,A.$

For new steady state armature ct be I_{a2} so the new speed n_2.

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$$T_{a1} = T_{a2}$$

$$k_t I_{sh1} I_{a1} = k_t I_{sh2} I_{a2}$$

$$I_{sh1} I_{a1} = I_{sh2} I_{a2}$$

$$1 \times 19 = 0.95 \, I_{a2}$$

$$I_{a2} = \frac{19}{0.95}$$

$$\boxed{I_{a2} = 20\,A}$$

To calculate new speed, we have to calculate new back emf.

$$E_{b2} = k_g I_{sh2} n_2$$

$$= k_g \times 0.95 n_2$$

$$= 220 - 20 \times 0.5 = 210V$$

$$\therefore \frac{k_g \times 0.95 \times n_2}{k_g \times 1 \times 1000} = \frac{210.5}{210}$$

$$n_2 = \frac{210.5}{210} \times \frac{1000}{0.95}$$

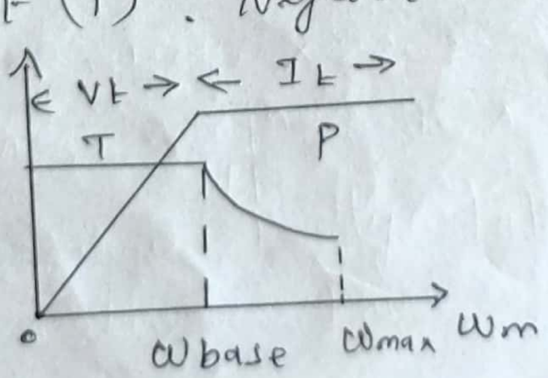
$$\therefore \boxed{n_2 = 1055.14\ rpm}$$

Scanned with CamScanner

A variable speed drive system uses a DC motor that is supplied from a variable-voltage source. The torque and power profiles are shown in fig. The drive speed is varied from 0 to 1500 rpm (base speed) by varying the terminal voltage from 0 to 500V with the field current maintained constant.

(i) Determine the motor armature current if the torque is held constant at 300 Nm up to the base speed.

(ii) The speed beyond the base speed is obtained by field weakening while the armature voltage is held constant at 500V. Determine the torque available at a speed of 3000 rpm if the armature current is held constant at the value obtained in part (i). Neglect all losses.



(Nov/Dec 2018)

soln:-

(a) $N_b = 1500$ rpm, $V_t = 500V = E_a$

$$k_a\phi = \frac{500}{1500 \times 2\pi/60} = 3.1831$$

$N \propto \frac{E_b}{\phi}$

$E_b = \phi \frac{2P}{60A} N$

$E_b = k\phi N$

$k\phi = \frac{E_b}{N}$

$$I_a = \frac{T}{k_a\phi} = \frac{300}{3.1831} = 94.2477A$$

(b) $n = 3000$ rpm, $V_t = E_a = 500V$

$$k_a\phi = \frac{500}{3000 \times 2\pi/60} = 1.5916$$

$$T = 1.5916 \times 94.2477$$

$$T = 150 \text{ N.m}$$

(or) $$T = \frac{P}{\omega_m} = \frac{500 \times 94.2477}{3000 \times 2\pi/60}$$

$$\boxed{T = 150 \text{ N-m}}$$

Scanned with CamScanner

Example 3: A 220V, DC shunt motor with an armature resistance of 0.4 Ω and a field resistance of 110 Ω drives a load, the torque of which remain constant. The motor draws from the supply, a line current of 32 A when the speed is 450 rpm. If the speed is to be raised to 700 rpm the what change must be effected in the value of the shunt field circuit resistance. Assume that the magnetization characteristics of the motor in a straight line.

Given data

supply voltage $V = 220V$

Armature resistance $Ra = 0.4\ \Omega$

Shunt field resistance $Rsh = 110\ \Omega$

speed $N_1 = 450$ rpm

Line current $I_L = 32$ A

speed $N_2 = 700$ rpm

To Find

shunt field circuit resistance.

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Now back emf $E_{b2} = V - I_{a2} R_a$

$= 500 - 28.75 \times 1.5$

$= 456.875 \text{ V}$

Using the relation

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\varphi_1}{\varphi_2} \quad [\because \varphi_1 = \varphi_2]$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

$$\frac{N_2}{1000} = \frac{456.875}{494.375}$$

$$\boxed{N_2 = 924.14 \text{ rpm}}$$

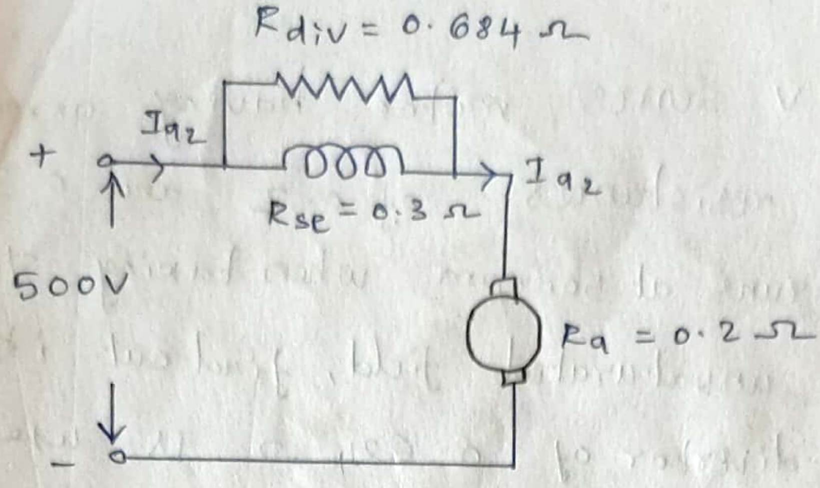
ii) shunt field reduced by 15 %

ie $\varphi_2 = 0.85 \varphi_2$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\varphi_1}{\varphi_2}$$

$$\frac{N_2}{1000} = \frac{456.875}{494.375} \times \frac{\varphi_1}{0.85 \varphi_1}$$

$$\boxed{N_2 = 1087.23 \text{ rpm}}$$



Since the torque remains constant

$$\phi_1 I_{a_1} = \phi_2 I_{a2}$$

$\phi \propto$ current through series field

$\phi_1 \propto I_{a_1}$

Current through the series field when a divertor is connected

$$= I_{a2} \times \frac{R_{div}}{R_{div} + R_{se}}$$

$$= I_{a2} \times \frac{0.684}{0.684 + 0.3}$$

$$= 0.695\ I_{a2}$$

Flux in this case $\phi_2 \propto 0.695\ I_{a2}$

$$I_{a_1}^2 = 0.695\ I_{a2}^2$$

$$I_{a2}^2 = \frac{I_{a_1}^2}{0.695} = \frac{70^2}{0.695}$$

$$I_{a2} = 83.96\ A$$

Scanned with CamScanner

Example 1. A 500 V series motor having armature and field resistances of 0.2 and 0.3 ohm respectively runs at 500 rpm when taking 70 amps. Assuming unsaturated field, find out its speed when field divertor of 0.684 Ω is used constant torque load.

Given data :

Supply voltage $V = 500V$

Armature resistance $R_a = 0.2 \Omega$

Series field Resistance $R_{se} = 0.3 \Omega$

Armature current $I_{a1} = 70 A$

speed $N_1 = 500$ rpm

Field divertor resistance $R_{div} = 0.684 \Omega$

To Find

speed N_2

solution:-

Back emf $E_{b1} = V - I_{a1}(R_a + R_{se})$

$= 500 - 70(0.2 + 0.3)$

$= 465 V$

Let I_{a2} be the current taken and ϕ_2 be the flux produced when a divertor is connected across the series field.

Scanned with CamScanner

Given data:

supply voltage $V = 500V$

Armature resistance $Ra = 1.5\Omega$

shunt field resistance $Rsh = 400\Omega$

No load current $I_0 = 5A$

No load speed $N_1 = 1000rpm$

Load current $I_L = 30A$.

To find:

i) speed at 30A

ii) speed at this load if the shunt field is reduced by 15% ie $\varphi_2 = 0.85\varphi_1$

Solution:-

shunt field current $I_{sh} = \frac{V}{Rsh} = \frac{500}{400}$

$= 1.25A$

No load armature current $I_{a0} = I_0 - I_{sh}$

$= 5 - 1.25 = 3.75A$

No load back emf $Eb_1 = V - I_{a0} Ra$

$= 500 - 3.75 \times 1.5$

$Eb_1 = 494.375V$

Load current $I_L = 30A$

Load armature current $I_{a2} = I_L - I_{sh}$

$= 30 - 1.25$

$= 28.75A$

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series field current $I_{se} = 0.695\, I_{a2}$

$= 0.695 \times 83.96$

$= 58.35\, A$

Back emf $E_{b2} = V - I_{a2} R_a - I_{se} R_{se}$

$= 500 - 83.96 \times 0.2 -$

58.35×0.3

$= 465.703\, V$

Using the relation

$$\frac{N_2}{N_1} = \frac{E_{b1}}{E_{b2}} \times \frac{\varphi_1}{\varphi_2}$$

$$\frac{N_2}{500} = \frac{465.703}{465} \times \frac{70}{83.96}$$

$$\boxed{N_2 = 418\ rpm}$$

Example 2: A 500 V dc shunt motor has armature and field resistances of 1.5 Ω and 400 Ω respectively. When running on no load the current taken is 5 A and the speed is 1000 rpm. Calculate the speed when motor is fully loaded and the total current drawn from the supply is 30 A. Also estimate the speed at this load if the shunt field current is reduced by 15%.

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Solution

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\varphi_1}{\varphi_2}$$

Since Magnetic ckt is unsaturated, it means that flux is directly proportional to the shunt current.

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}}$$

Since the motor is driving at load of constant torque,

$$T \propto \varphi_1 I_{a1} \propto \varphi_2 I_{a2}$$

$$\varphi_1 I_{a1} = \varphi_2 I_{a2}$$

$$I_{a2} = \frac{\varphi_1}{\varphi_2} \times I_{a1}$$

$$I_{sh1} = \frac{V}{R_{sh1}} = \frac{220}{110} = 2\,A$$

$$I_{sh2} = \frac{V}{R_{shT}} = \frac{220}{R_{shT}}$$

$R_{shT} \to$ Total resistance

Armature current $I_{a1} = I_L - I_{sh}$

$$= 32 - 2 = 30\,A$$

$$I_{a2} = 30 \times \frac{2}{\frac{220}{R_{shT}}} = 0.272\,R_{shT}$$

$E_{b1} = V - I_{a1} R_a$

$= 220 - 30 \times 0.4 = 208\ V$

$E_{b2} = V - I_{a2} R_a$

$= 220 - (0.272 R_{shT}) \times 0.4$

$= 220 - 0.1088 R_{shT}$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}}$$

$$\frac{700}{450} = \frac{220 - 0.1088 R_{shT}}{208} \times \frac{2}{220/R_{shT}}$$

$R_{shT} = 177.35\ \Omega$

$R_{sh1} + R_e = 177.35$

$R_e = 177.35 - 110$

$$\boxed{R_e = 67.35\ \Omega}$$

Scanned with CamScanner

A 220V shunt motor has armature and field resistance of 0.2 Ω and 220 Ω respectively. The motor is driving a constant load torque and running at 1000 rpm drawing 10A ct from the supply. calculate the new speed and armature current if an external armature resistance of value 5 Ω is inserted in the armature circuit. neglect armature reaction and saturation.

(April/May 2019)

soln:- For initial operating point

$I_{L_1} = 10\ A$, $r_a = 0.2\ \Omega$, $V = 220V$

$I_{sh_1} = 220/220 = 1\ A$

$I_{a_1} = 10 - 1 = 9\ A$

$T_{a_1} = k\ I_{sh_1} \cdot I_{a_1} = k \times 1 \times 9 = T_L$

$E_{b_1} = k_g\ I_{sh_1} \cdot n = k_g \times 1 \times 1000$

$= V - I_{a_1} \cdot r_a = 220 - 9 \times 0.2 = 218.2V$

$k_g \times 1 \times 1000 = 218 \cdot 2V.$

$I_{sh_1} = I_{sh_2} = 1\ A.$

$T_{a_2} = k_t \times 1 \times I_{a_2} = T_L$

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$$E_{b2} = k_g \times 1 \times n_2$$

$$= V - I_{a2}(r_a + R_{ext})$$

$$\therefore k_g \times 1 \times n_2 = 220 - I_{a2} \times 5.2$$

Taking ratios

$$\frac{T_{a2}}{T_{a1}} = \frac{k_t \times 1 \times I_{a2}}{k_t \times 1 \times I_{a1}}$$

$$\frac{T_{a2}}{T_{a1}} = \frac{k_t \times 1 \times I_{a2}}{k_t \times 1 \times 9}$$

$$\frac{E_{b2}}{E_{b1}} = \frac{k_g \times 1 \times n_2}{k_g \times 1 \times 1000} = \frac{220 - I_{a2} \times 5.2}{218.2}$$

$$\frac{n_2}{1000} = \frac{220 - 9 \times 5.2}{218.2}$$

or $$\frac{n_2}{1000} = \frac{173.2}{218.2}$$

$$n_2 = \frac{173.2 \times 1000}{218.2}$$

$$\boxed{\therefore n_2 = 793.76 \text{ rpm}}$$

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EXAMPLE A 15 kW, 250 V, 1200 rpm, shunt motor has 4 poles, 4 parallel armature paths and 900 armature conductors. Assume $R_a = 0.2\ \Omega$. At rated speed and rated output the armature current is 75 A, and $I_{sh} = 1.5$ A, calculate,

1. The flux / pole
2. The torque developed
3. Rotational losses
4. Efficiency
5. The shaft load
6. If the shaft load remains fixed, but the field flux is reduced to 70% of its value by field control, determine the new operating speed.

Solution

1. $E_b = V - I_a R_a = 250 - 75 \times 0.2$

$E_b = 235$ V

$$E_b = \frac{\varphi P Z N}{60 A}$$

$$\varphi = \frac{E_b \times 60 \times A}{P Z N}$$

$$= \frac{235 \times 60 \times 4}{4 \times 900 \times 1200}$$

$\boxed{\varphi = 13.05}$ mWb

2. $T = 9.55 \times \frac{E_b I_a}{N}$

$= 9.55 \times \frac{235 \times 75}{1200}$

$T_a = 140.26$ N-m

3. power developed in armature,

$= E_b I_a$

$= 235 \times 75 = 17625$ W

Rotational losses $= E_b I_a - P_{out}$

$= 17625 - 15000$

$= 2625$ W

4. Efficiency

I/p power $P_{in} = V I_L$

$= V(I_a + I_{sh})$

$= 250(75 + 1.5)$

$= 19125$ W

$\eta = \frac{P_{out}}{P_{in}} \times 100$

$= \frac{15000}{19125} \times 100$

$\boxed{\eta = 78.43\ \%}$

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5. shaft Load

$$T_{sh} = 9.55 \frac{P_{out}}{N}$$

$$= 9.55 \times \frac{15000}{1200}$$

$$T_{sh} = 119.37 \text{ N-m}$$

6. speed (N_2)

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

Now $T_a \propto \phi I_a$

$$T_{a1} \propto \phi_1 I_{a1}$$

$$T_{a2} \propto \phi_2 I_{a2}$$

$$\phi_1 I_{a1} = \phi_2 I_{a2}$$

$$\phi_1 75 = 0.7 \phi_1 I_{a2}$$

$$I_{a2} = \frac{\phi_1 \times 75}{0.7 \times \phi_1}$$

$$I_{a2} = 107.14 \text{ A}$$

$$E_{b2} = V - I_{a2} R_a$$

$$= 250 - 107.14 \times 0.2$$

$$\boxed{E_{b2} = 228.57 \text{ V}}$$

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$$\frac{N_2}{1200} = \frac{228.57}{235} \times \frac{\phi_1}{0.741}$$

$$\boxed{N_2 = 1667.3} \text{ rpm}$$

H.W

A Dc shunt machine while running as a generator at a voltage of 250V, at 1000 rpm on no load it has armature resistance of 0.5 Ω and field resistance of 250 Ω. when the machine runs as motor, input to it at no load is 3 A at 250 V. calculate the speed and efficiency of the machine when it runs as motor taking 40 A at 250 V. Armature reaction weakens the field by 4 %.

Ans: $N_2 = 964.27$ rpm

No load i/p $P_{no\ load} = 750$ W

constant loss = 748 W

Total loss at full load = 1508.5 W

$P_{in_{Load}} = 10000$ W, $P_{out} = 8491.5$ W

$\eta = 84.91\%$

Applications:

1. Shunt motors may be used for driving centrifugal pumps and light machine tools, wood working machines, lathe etc.

2. Series motors are used for electric trains, cranes, hoists, fans, blowers, conveyers, lifts etc.

3. Compound machines are used for driving heavy machine tools, punching machines etc.

SPEED CONTROL OF DC MOTOR.

The speed of DC motor is given by,

$$N = \frac{V - I_a R_a}{2\varphi}\left(\frac{A}{P}\right)$$

(or) $$N = \frac{K\,(V - I_a R_a)}{\varphi}$$

where, V = applied voltage

I_a = armature current

R_a = armature resistance

φ = flux per pole

K = constant.

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Three methods of speed controls

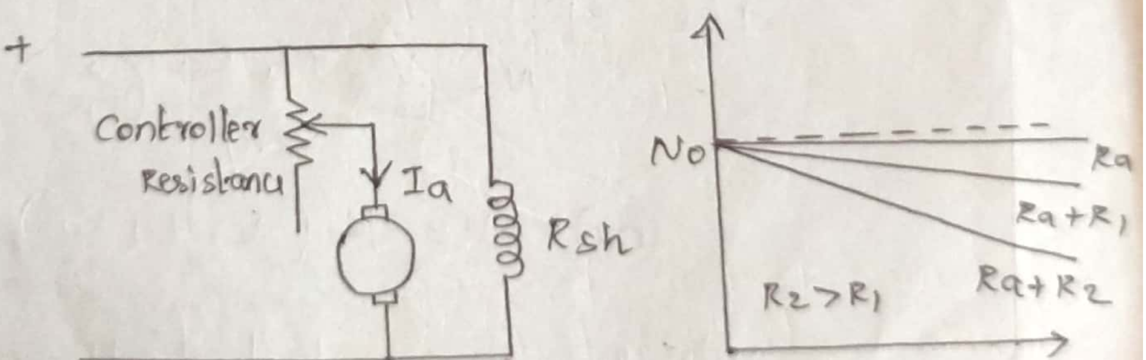
1. By varying the resistance in the armature circuit.
2. By varying the flux
3. By varying the applied voltage.

1. By varying the resistance in the armature circuit.

A variable resistance R is connected in series with armature circuit.

Here the input voltage V is constant. The speed of the motor can be controlled by varying the resistor R. so the speed equation becomes.

$$N = \frac{K\left[V - I_a(R_a + R)\right]}{\phi}$$



Advantages

- Simple method of speed control.

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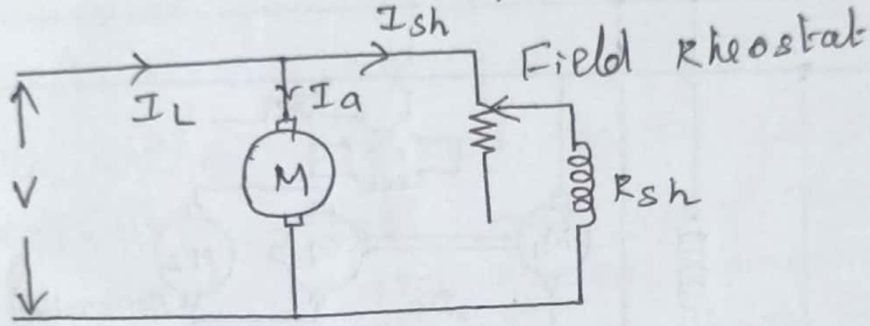
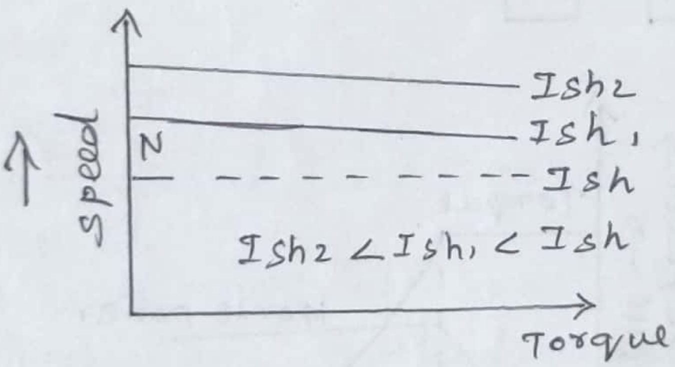
disadvantage

1. More power consumption
2. The change in speed with the change in load becomes large.

2. By varying the flux

The speed is inversely proportional to flux i.e. $N \propto 1/\phi$





The flux of a DC motor can be changed by changing the field current.

The motor speed canbe increased by decreasing the flux.

This method of speed control can be used for above rated speed control method.

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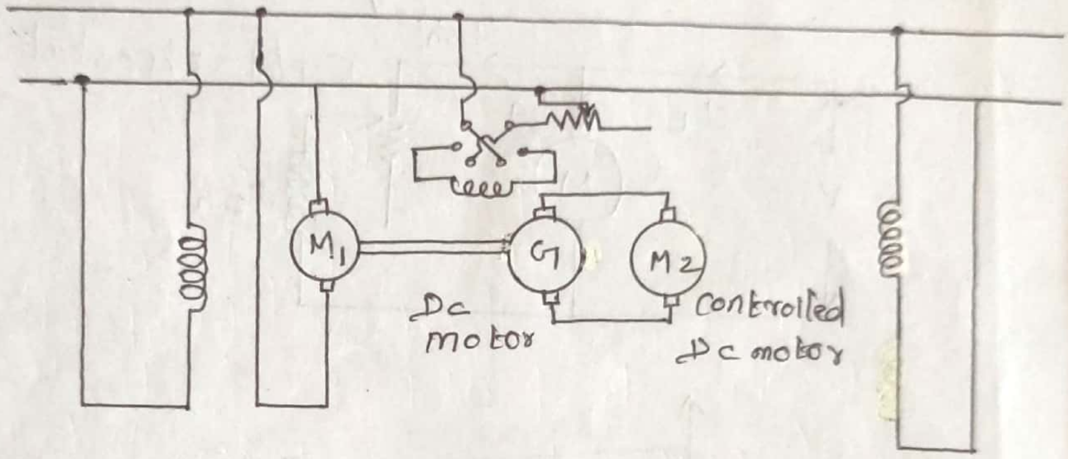
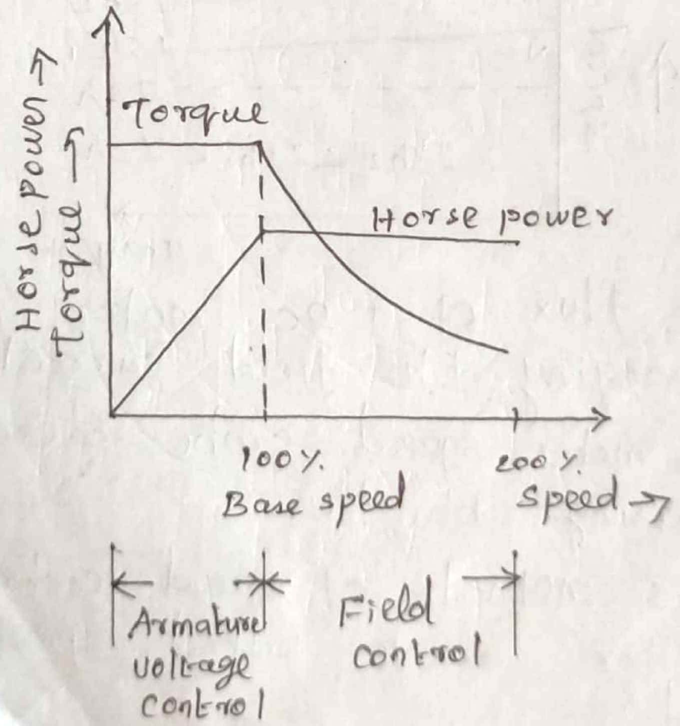
Ward Leonard control system.

The ward Leonard speed control system is shown in figure. This system is mainly used where very sensitive speed is required as for electric excavators, elevators, colliery winders and the main drives in steel mills and paper mills.





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→ It consists of three DC machines, ie two DC motors and one DC generator.

→ The motor-Generator set runs at constant speed. The voltage of the generator can be varied from zero to maximum value by means of its field regulator.

→ The generated dc voltage is fed to the controlled dc motor.

→ The direction of rotation of the controlled dc motor M2 can be changed by reversing the direction of the field current of generator.

→ This method of speed control is combination of armature control and flux control.

→ The ward Leonard system provides a constant torque as well as constant horse power drive.

Advantages:

1. Full forward and reverse speed can be achieved.
2. A wide range of speed control is possible.
3. Short-time overload capacity is large.

Disadvantage.

1. High Initial cost
2. costly
3. The drive produces noise
4. It require frequent maintenance.

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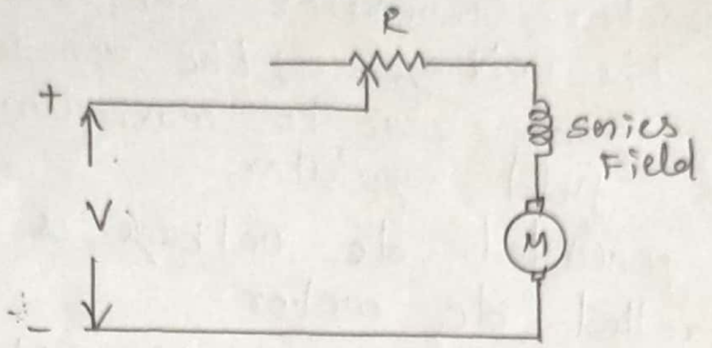
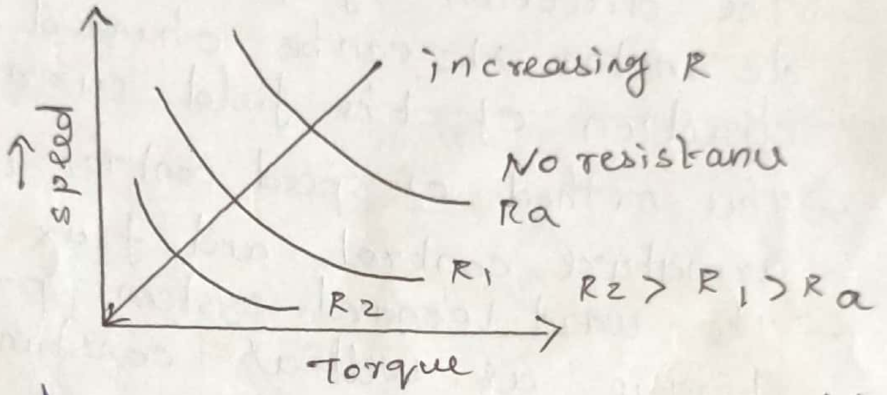
speed control of DC series motor.

Variable resistance in series with motor.





By reducing the voltage across the armature, the motor speed also decreases. Because the applied voltage is directly proportional to the speed, $N \propto E_b$

$$E_b \propto V$$

$$\therefore N \propto V$$

Flux control method.

i) Field divertor method.

Field divertor means, a variable resistance is connected across the series field winding. By varying the resistance, the current flowing through the series field changes. Due to decrease in field current, the motor speed also increases.

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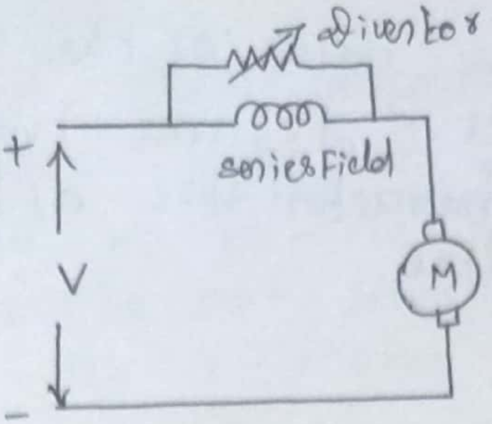
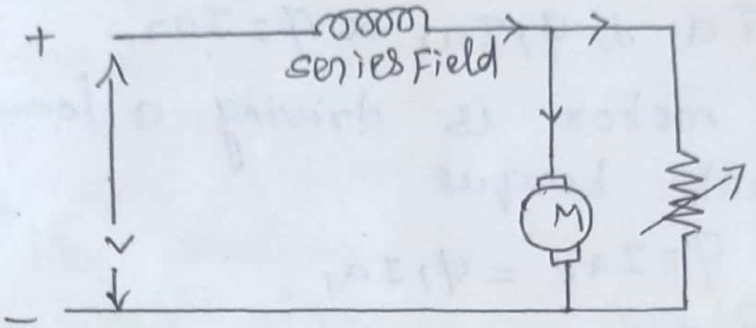


(ii) Armature divertor method.

Here, a variable resistance is connected across the armature as shown in figure.



This method of control gives speeds lower than the normal speed.

Due to current increase, series field flux also increases. The the speed of the motor can be decreased.

Ex: A 220V, DC shunt motor with an armature resistance of 0.4 Ω and a field resistance of 110 Ω drives a load, the torque of which remains constant. The motor draws from the supply a line current of 32 A when the speed is 450 rpm. If the speed is to be raised to 700 rpm what change must be effected in the value of the shunt field circuit resistance? Assume that the magnetization characteristic of the motor is a straight line.

Soln:-

Formulas used

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$$T_a \propto \phi_1 I_{a1} \propto \phi_2 I_{a2}$$

Since motor is driving a load of constant torque.

$$\phi_2 I_{a2} = \phi_1 I_{a1}$$

(or) $I_{sh2} I_{a2} = I_{sh} I_{a1}$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2\,A$$

$$I_{sh2} = \frac{220}{R_t}$$

where R_t = Total resistance of the shunt field circuit,

$$I_{a1} = I_L - I_{sh1} = 32 - 2 = 30\,A$$

$$I_{a2} = I_{a1} \times \frac{I_{sh1}}{I_{sh2}} = 30 \times \frac{2}{220/R_t}$$

$$E_{b2} = V - I_{a2} \cdot R_a = 220 - (0.272 R_t \times 0.4)$$

$$= 220 - 0.1088\, R_t$$

$$E_{b1} = V - I_{a1} \cdot R_a = 220 - 30 \times 0.4 = 208\,V$$

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UNIT- IV

SINGLE PHASE TRANSFORMER.

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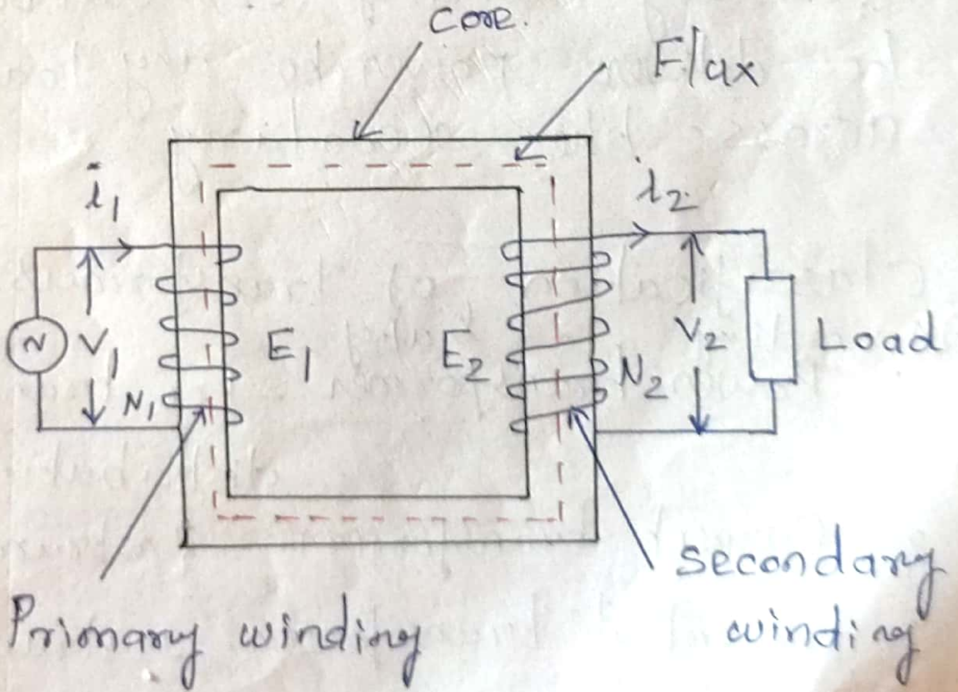
T-3

CONSTRUCTION AND PRINCIPLE OF OPERATION.

Introduction:

Transforms works on the principle of electromagnetic induction.

A transformer is an electrical device, having no moving parts which transfer electricity from one circuit to another at the same fequeny.



working Principle:

When the primary winding is connected to an ac source an exciting current flows through the winding. As the current is alternating, it will produce an alternating flux in the core which will be linked by both the primary and secondary windings.

The induced emf in the primary winding (E_1) is almost equal to the applied voltage V_1 and will oppose the applied voltage. The emf induced in the secondary winding (E_2) can be utilised to deliver power to any load connected across the secondary.

Classification of Transformers.

(i) According to Duty

1. Power transformer - For transmission and distribution process.
2. Current transformer - Instrument transformer
3. Potential transformer - Instrument transformer.

(ii) Construction

1. core type 2. shell type 3. Berry type.

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(iii) Voltage output

1. step down transformer
2. step up transformer
3. Auto transformer.

(iv) Application

1. welding transformer
2. Furnace transformer

(v) Cooling

1. Duct type
2. oil immersed

(vi) Input supply

1. single phase transformer
2. Three phase transformer.

Construction:

The main component of a transformer are,

i) the magnetic core
ii) Primary and secondary winding
iii) Insulations of windings.
iv) Expansion tank or Conservator.
v) Terminals and Insulators
vi) Tank, oil, cooling arrangement

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vii) Temperature gauge, oil gauge.
viii) Buchholz relay
ix) Breather.

Magnetic core:

- Magnatic circuits consists of an iron core
- Lamination mad up of silicon steel.
- Thickness of laminations from 0.35mm to 0.5mm.
- Laminations are insulated by thin coat of varnish.

Two types of transformer cores are:

a. Core type
b. Shell type.

a. Core type:

Here the windings surrounds a considerable part of core.

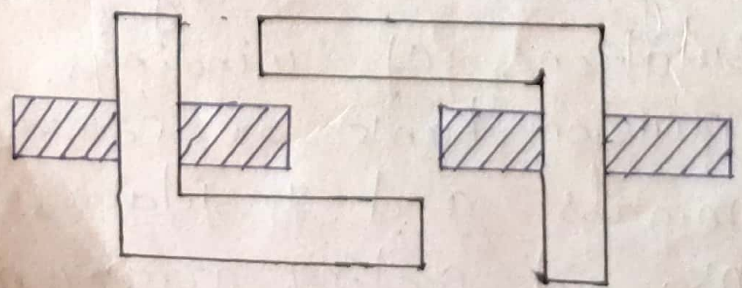

Fig: core Type.

(b) shell Type

- Here the core surrounds the considerable part of windings.

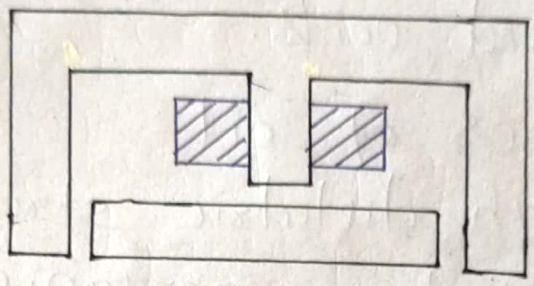

Fig: shell type

winding

- There are two windings
  1. Primary
  2. secondary
- winding are made up of copper.

Insulation:

- Paper is used as the basic conductor insulation.
- Enamel insulation is used as the interturn insulation for low voltage transformers.
- For power transformer uses enamelled copper with paper insulation.

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Insulating oil

The oil used in the transformer protects the paper from dirt and moisture and removes the heat produced in the core and coils.

Properties of oil:-

1. High dielectric strength
2. Free from inorganic acid
3. Low viscosity
4. Good resistance to emulsion

Expansion tank or conservator:

- A small auxiliary oil tank may be mounted above the transformer and connected to main tank by a pipe.
- It function is to keep the transformer tank full of oil despite expansion or contraction of the coil with the changes in temperature.

Temperature Gauge:

- Indicate hot oil or hottest spot temperature.

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Oil Gauge

To indicate the oil level present inside the tank.

Buchholz Relay:

It will give an alarm in case of minor fault and to disconnect the transformer from the supply mains in case of severe faults.

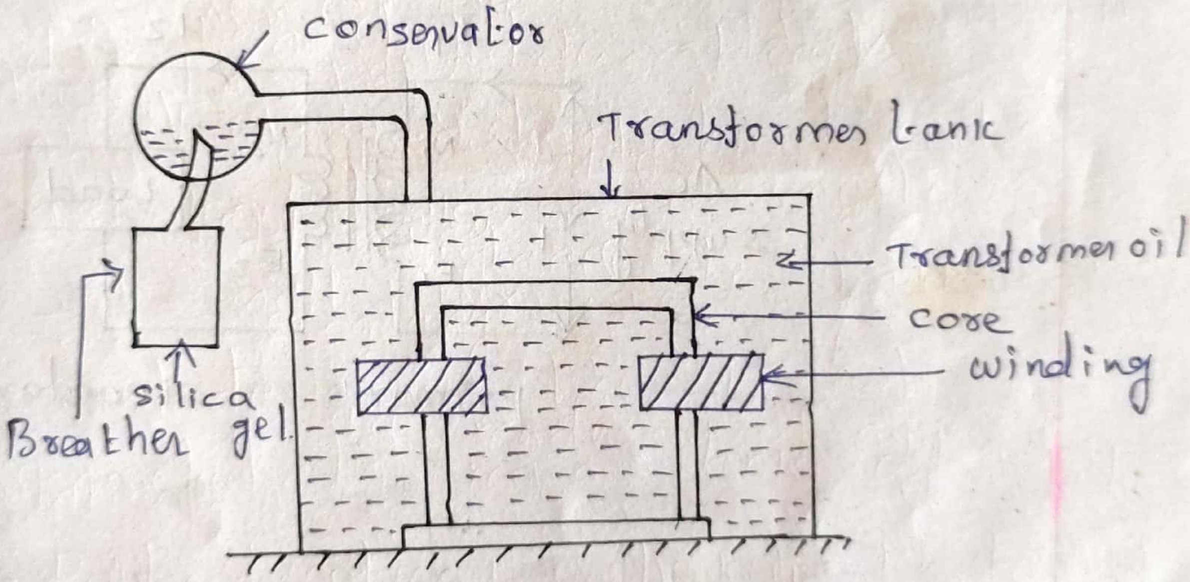

Fig: construction of Transformer.

Cooling arrangement:

a) oil immersed natural cooled
b) oil immersed forced air cooled
c) Oil immersed water cooled
d) oil immersed forced oil cooled

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EMF equation of a transformer.

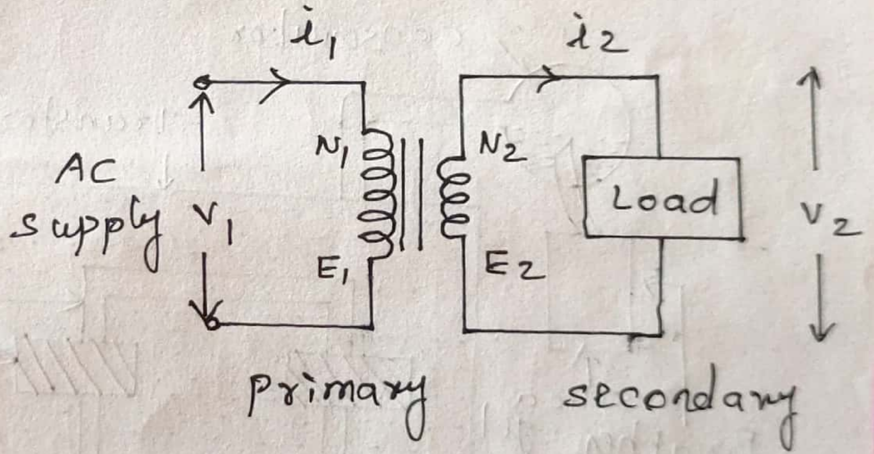
Consider a transformer arrangement,

N_1 - Number of primary turns

N_2 - Number of secondary turns

ϕ_m - Maximum value of flux in the core in wb.

B_m - Maximum value of flux density in the core in wb/m^2.



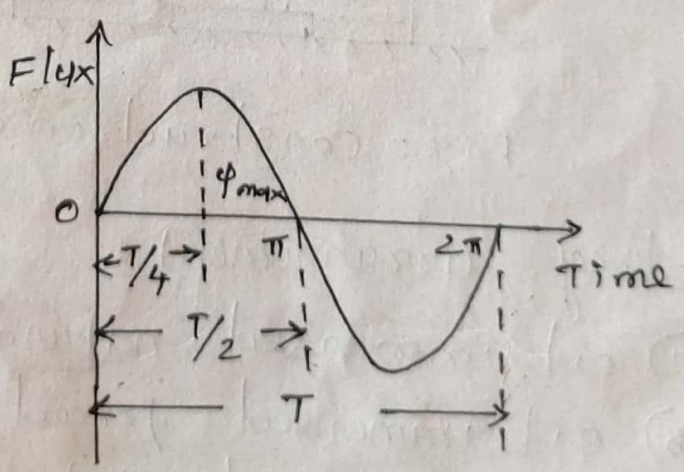


Fig: The Alternating flux established in a transformer.

A - Area of the core in m^2.

f - Frequency of the Ac supply in Hertz.

V_1 - Supply voltage across primary in volts.

V_2 - Terminal voltage across secondary in volts.

I_1 - Full load primary current in amperes

I_2 - Full load secondary current in amperes.

E_1 - Emf induced in the primary in volts.

E_2 - Emf induced in the secondary in volts.

we know that $T = 1/f$

where f is the frequency in Hz.

∴ Average rate of change of flux

$$= \frac{\phi m}{1/4f} \text{ wb/seconds.}$$

the average value of emf induced / turn

$$= 4f \times \phi m \text{ volt} \quad (1)$$

$$\text{Form factor} = \frac{\text{R.M.S value}}{\text{Average value}}$$

$$= 1.11$$

∴ RMS value = Form factor × Average value

∴ RMS value of emf induced / turn

$$= 1.11 \times 4f \times \phi m$$

$$= 4.44 f \phi m \text{ volts.} \quad \text{--- (2)}$$

$$\phi m = Bm \cdot A$$

∴ RMS value of emf induced in the entire primary winding.

$$E_1 = 4.44 f \phi m \times N_1$$

or

$$E_1 = 4.44 f Bm A N_1 \text{ volts.} \quad \text{--- (3)}$$

Similarly $E_2 = 4.44 f Bm A N_2$ volts. --- (4)

Transformation ratio.

For an ideal transformer,

$V_1 = E_1$, $V_2 = E_2$ and

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2}; \quad \frac{E_2}{E_1} = \frac{I_1}{I_2} \quad \text{--- (5)}$$

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From equation 3 and 4,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \quad \text{——— (6)}$$

From eqn 5 and 6,

we have $\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = k$ ——— (7)

where k is called transformation ratio.

when $N_2 > N_1$ ie $k > 1$ – step up transformer

$N_2 < N_1$ ie $k < 1$ – step down transformer.

Ex: A single phase transformer has 500 primary and 1200 secondary turns. Net cross sectional area of the core is 80 cm^2. If the primary winding is connected to 50 Hz supply at 500V, calculate the value of maximum flux density on core and the emf induced in the secondary.

Given data :

$N_1 = 500$ turns $\quad A = 80\ cm^2$

$N_2 = 1200$ turns $\quad = 80 \times 10^{-4}\ m^2$

$f = 50\ Hz \quad E_1 = 500V$

Solution :-

$$E_1 = 4.44 f \phi_m N_1$$

$$\phi_m = \frac{E_1}{4.44 f N_1} = \frac{500}{4.44 \times 50 \times 500}$$

$$= 4.5\ mwb.$$

$$B_m = \phi_m / A = \frac{4.5 \times 10^{-3}}{80 \times 10^{-4}}$$

$$= 0.563\ wb/m^2.$$

secondary induced emf (E_2)

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$E_2 = E_1 \cdot \frac{N_2}{N_1}$$

$$= 500 \times \frac{1200}{500}$$

$$\boxed{E_2 = 1200V}$$

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Ex: calculate the flux in the core of a single phase transformer having a primary voltage of 230V, at 50Hz and 50 turns. If the flux density in the core is one tesla, calculate the net cross sectional area of the core.

Given data:

$V_1 = E_1 = 230V$, $f = 50Hz$, $N_1 = 50$ turns,

$B_m = 1$ tesla.

solution:

E.M.F induced in the primary winding.

$$E_1 = 4.44 f \phi_m N_1$$

$$\phi_m = \frac{E_1}{4.44 f N_1}$$

$$= \frac{230}{4.44 \times 50 \times 50}$$

$$= 0.0207 \text{ wb}$$

$$\phi_m = B_m A$$

$$A = \phi_m / B_m = \frac{0.0207}{1}$$

$$\boxed{A = 0.0207} \ m^2$$

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Equivalent circuit of a Transformer.

An equivalent circuit is merely a circuit interpretation of the equations which describe the behaviour of the system. Fig shows the equivalent circuit of a transformer.

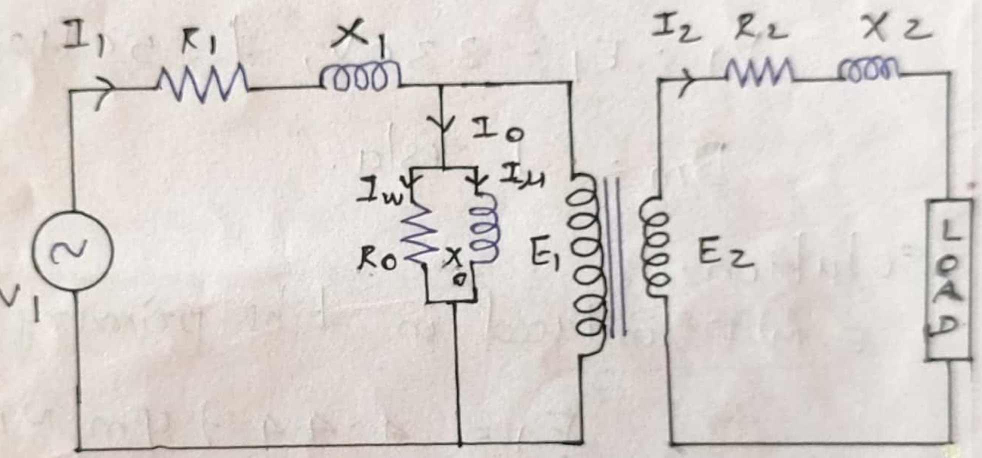


Fig: Equivalent circuit of Transformer.

In this equivalent circuit

R_1, X_1 - Primary winding resistance and reactance in Ω.

R_0 - No load resistance in Ω.

X_0 - No load reactance in Ω

I_1 - Full load primary current in A.

I_0 - No load primary current in A

I_2' - Load component of primary current in A.

I_w - Working component

I_μ - Magnetising component.

E_1 - Induced emf in primary winding in V

E_2 - Induced emf in secondary winding in V

R_2, X_2 - Secondary winding resistance and reactance in Ω.

Z_L - Load impedance in Ω

I_2 - Full load secondary current in A.

K - Transformation ratio.

Equivalent circuit of a transformer referred to primary.

If all the secondary parameters are transferred to the primary side, we get the equivalent circuit of transformer referred to primary as shown in fig.

When secondary referred to primary

- Resistance and reactances are divided by k^2
- voltages are divided by k
- currents are multiplied by k.

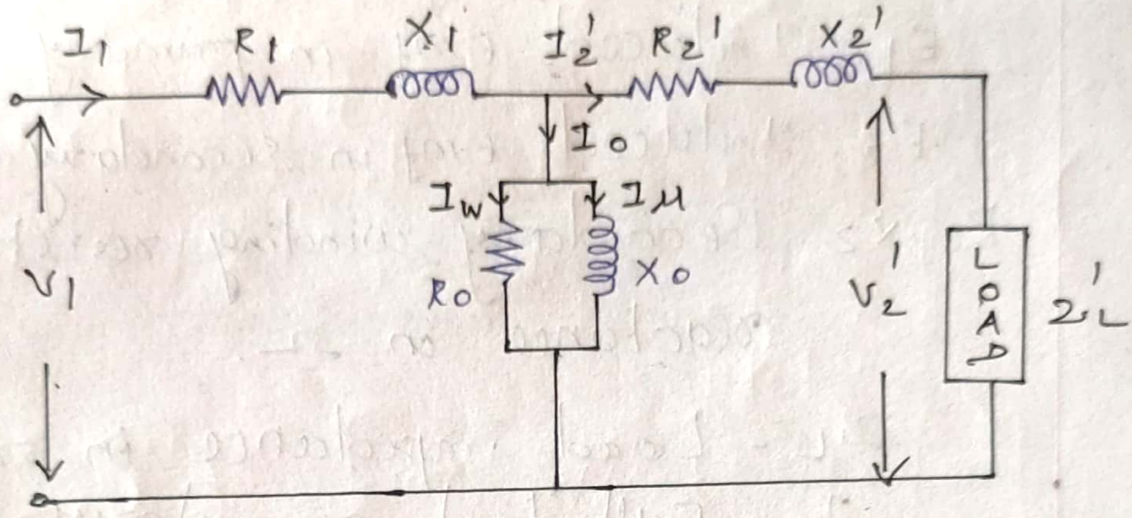


Fig: Exact equivalent circuit.

$$R_2' = \frac{R_2}{k^2} \qquad V_2' = \frac{V_2}{k}$$

$$X_2' = \frac{X_2}{k^2} \qquad R_0 = \frac{V_1}{I_w}$$

$$I_2' = kI_2 \qquad X_0 = \frac{V_1}{I_\mu}$$

$$Z_L' = \frac{Z_L}{k_2}$$

Approximate equivalent circuit.

- The no load current I_0 is only 1-3 % of rated primary current so I_2' is practically equal to I_1.
- Due to this equivalent circuit can be

simplified by transfering the exciting branch (R_0 and I_1) to the left position of the circuit as shown in fig.

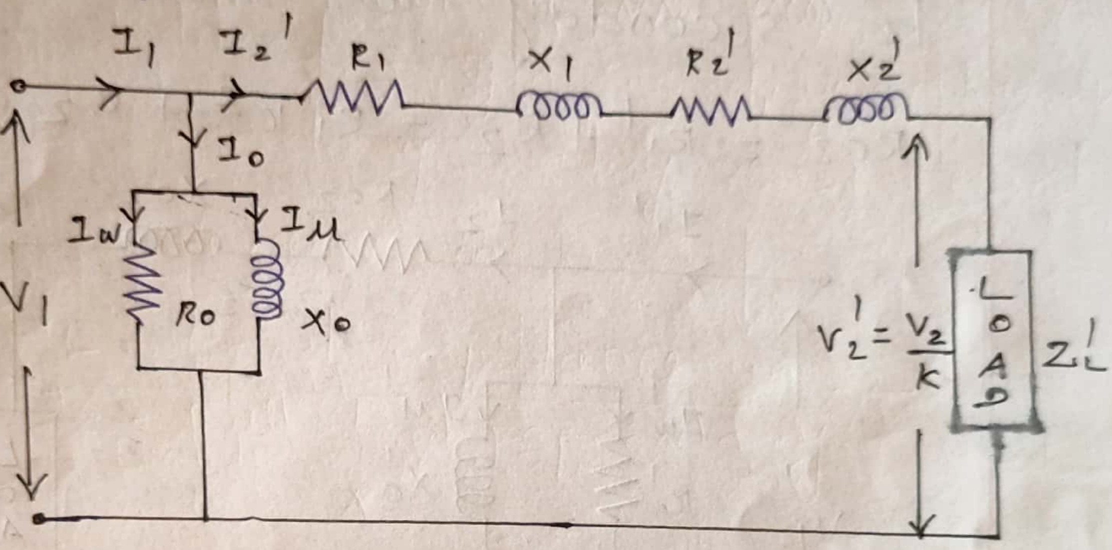
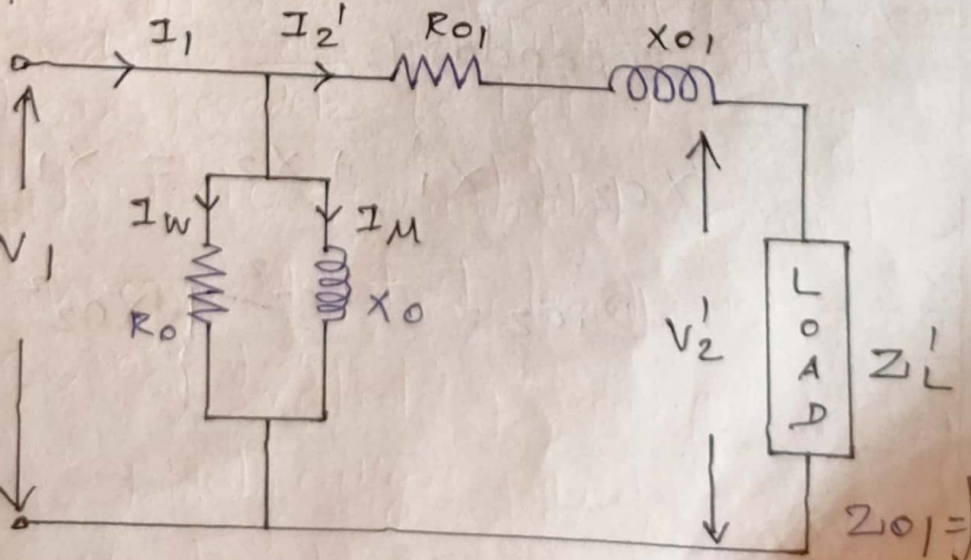


Fig: Simplified Equivalent circuit.

The below fig shows combined R_1 and R_2' and X_1 and X_2'. i.e $R_{01} = R_1 + R_2'$ and $X_{01} = X_1 + X_2'$



$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$

The above fig shows all parameters referred to primary. similarly the figure shows all parameters referred to secondary as shown below.

$$R_{02} = R_1' + R_2 = R_1 K^2 + R_2$$

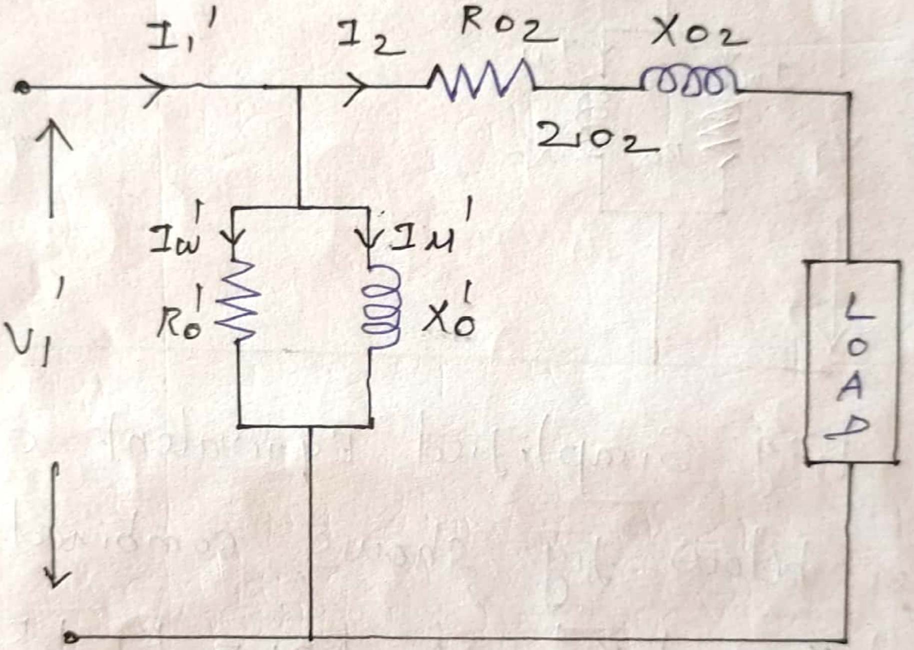


Fig: Equivalent circuit referred to secondary.

$$X_{02} = X_1' + X_2 = X_1 K^2 + X_2$$

$$Z_{02} = R_{02}^2 + X_{02}^2$$

Ex: A 2000/200 V transformer has primary resistance and reactance of 2 Ω and 4 Ω respectively. The corresponding secondary values are 0.025 Ω and 0.04 Ω. Determine (1) equivalent resistance and reactance of primary referred to secondary (2) total resistance and reactance referred to secondary (3) equivalent resistance and reactance of secondary referred to primary (4) total resistance and reactance referred to primary.

Given data

$V_1 = 2000V$, $V_2 = 200V$, $R_1 = 2\Omega$, $X_1 = 4\Omega$,

$R_2 = 0.025\Omega$, $X_2 = 0.04\Omega$

To Find

(1) R_1', X_1' (2) R_{01} (3) R_2', X_2' (4) X_{02}.

Soln:-

(i) Transformation ratio $K = \frac{V_2}{V_1} = \frac{200}{2000} = 0.1$

Equivalent resistance of primary referred to secondary,

$$R_1' = R_1 K^2 = 2 \times 0.1^2$$

$$R_1' = 0.02\Omega$$

Equivalent reactance of primary referred to secondary,

$$X_1' = X_1 K^2 = 4 \times 0.1^2 = 0.04\Omega$$

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(2) Total resistance referred to secondary,

$$R_{O2} = R_2 + R_1' = 0.025 + 0.02$$

$$R_{O2} = 0.045\ \Omega$$

Total reactance referred to secondary,

$$X_{O2} = X_2 + X_1' = 0.04 + 0.04$$

$$X_{O2} = 0.08\ \Omega$$

(3) Equivalent resistance of secondary referred to primary,

$$R_2' = \frac{R_2}{K^2} = \frac{0.025}{0.1^2}$$

$$R_2' = 2.5\ \Omega$$

Equivalent reactance of secondary referred to primary $X_2' = \frac{X_2}{K^2} = \frac{0.04}{0.1^2}$

$$X_2' = 4\ \Omega$$

(4) Total resistance referred to primary R_{O1}

$$R_{O1} = R_1 + R_2' = 2 + 2.5 = 4.5\ \Omega$$

Total reactance referred to primary

$$X_{O1} = X_1 + X_2'$$

$$= 4 + 4$$

$$\boxed{X_{O1} = 8\ \Omega}$$

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Phasor Diagram:

The following steps are followed to draw the phasor diagram for different power factor.

Step 1: Draw the flux vector φ. It acts as reference line OA.

Step 2: Draw the induced emf E_1 (OB). The angle between E_1 and φ is 90° lagging.

Step 3: Draw the $-E_1$ line. It is opposite to E_1 (OC).

Step 4: Draw the no load primary current I_0 (OD).

Step 5: Draw the secondary terminal voltage V_2 in a particular direction (OE).

Step 6: Draw secondary current I_2 vector (OF).

unity power factor: I_2 and V_2 are in phase

Lagging power factor: I_2 is lagging with respect to V_2 by an angle φ_2

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Leading power factor: I_2 is leading with respect to V_2 by an angle ϕ_2.

step 7: Draw I_2R_2 drop line. It is parallel to current vector I_2 (EG)

step 8: Draw I_2X_2 drop line. It is perpendicular to current vector I_2 (GH).

step 9: I_2X_2 line is joined with E_1 line. This is point E_2 (OH)

step 10: Draw I_2' line. It opposes I_2 ie 180° out of phase (OI).

step 11: Draw I_1 line (OJ) $\bar{I_1} = \bar{I_0} + \bar{I_2'}$

step 12: Draw the I_1R_1 drop line. It is parallel to current vector I_1 (CK).

step 13: Draw I_1X_1 drop line. It is perpendicular to current vector I_1 (KL).

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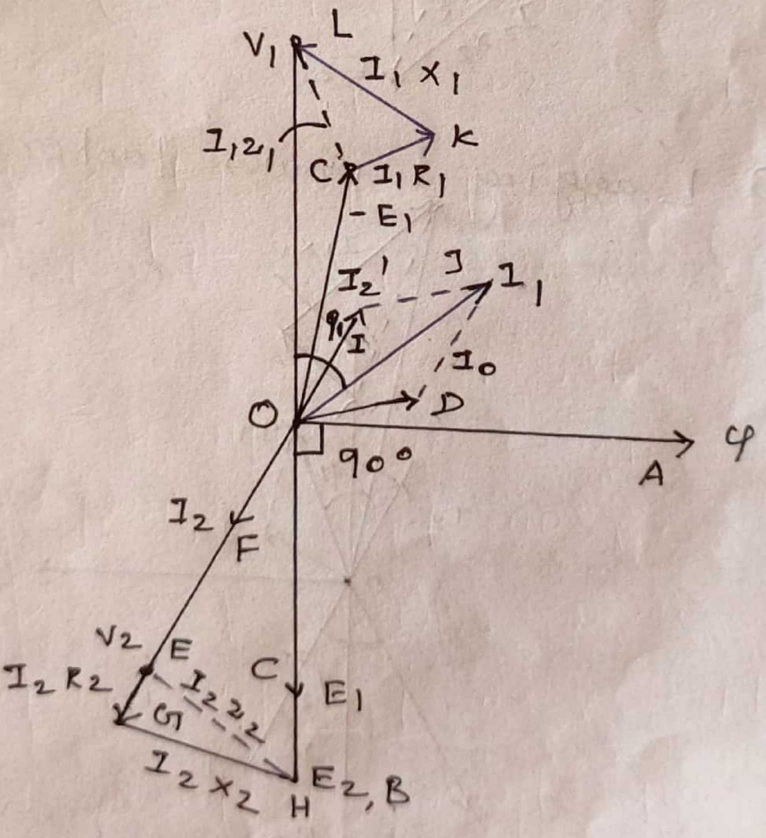
Step 14: Join I_1X_1 point and O. We can get V_1 (OL). The angle between V_1 and I_1 is ϕ_1.

Load power factor $= \cos\phi_2$

Primary power factor $= \cos\phi_1$

Input power to transformer $P_1 = V_1 I_1 \cos\phi_1$

Output power from transformer $P_2 = V_2 I_2 \cos\phi_2$.



(a) Unity power factor.

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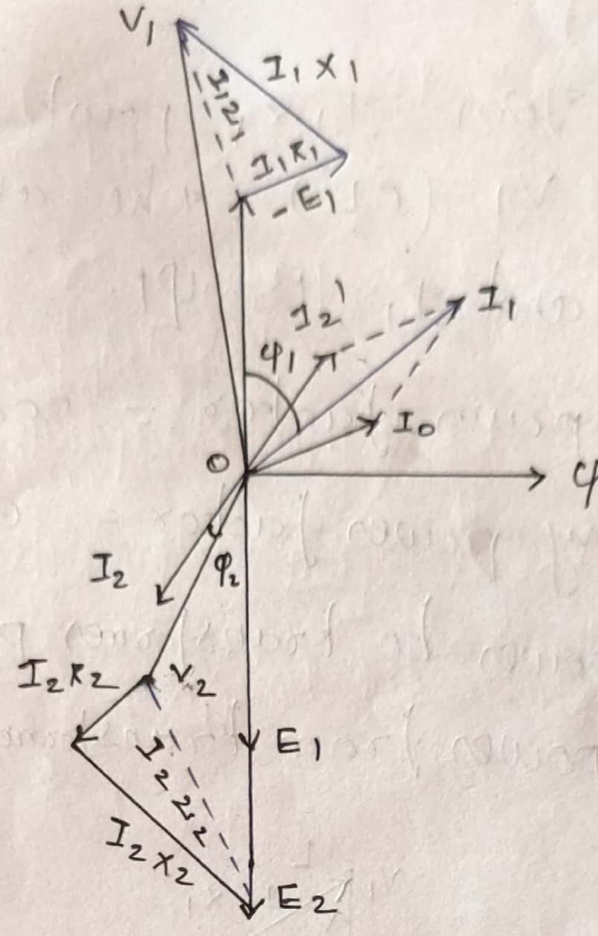
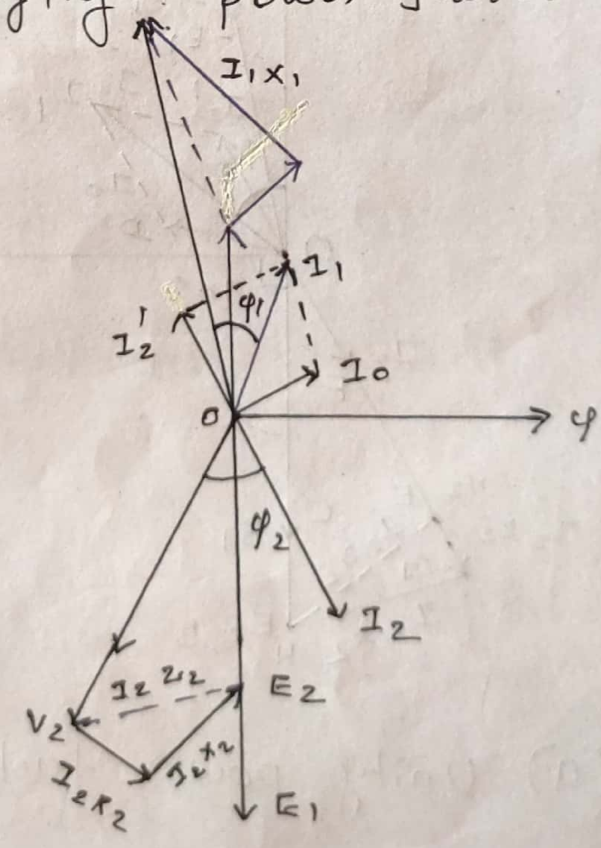


(b) Lagging power factor.



(c) Leading power factor.

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Losses in a Transformer.

In any transformer, there are no friction or windage losses. The losses occurring are,

i. copper loss

ii. Iron loss or core loss.

Copper loss

- This loss is due to be ohmic resistance of the transformer winding.

$$P_{copper\ Loss} = I_1^2 R_1 + I_2^2 R_2.$$

Iron Loss (or) Core Loss.

- Iron loss is caused by the alternating flux in the core and consists of hysteresis and eddy current loss.
- It is constant for all loads.

Hysteresis Loss $P_h = K_h B_{max}^{1.6} f.$

Eddy current loss $P_e = K_e B_{max}^2 f^2.$

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Efficiency of a Transformer:

Transformer efficiency $\eta = \frac{\text{Output power}}{\text{Input power}}$

$$\eta = \frac{\text{output power}}{\text{o/p power} + \text{Losses}}$$

Losses = Iron Loss + Copper loss

o/p power = $V_2 I_2 \cos\varphi$

where

V_2 - secondary terminal voltage on load.

I_2 - secondary current at load

$\cos\varphi$ - power factor of the load.

Iron loss $P_i = W_0$ - OC Test.

Copper loss $P_{cu} = W_s$ - SC Test.

Copper loss at a load n times the Full load $= n^2 P_{cu}$.

So, transformer efficiency $\eta = \frac{n V_2 I_2 \cos\varphi}{n V_2 I_2 \cos\varphi + P_i + n^2 P_{cu}}$

$$\frac{d}{dI_2}\left(V_2 \cos\phi_2 + \frac{P_i}{I_2} + I_2 R_{02}\right) = 0$$

$$0 - \frac{P_i}{I_2^2} + R_{02} = 0 \text{ (or)}$$

$$P_i = I_2^2 R_{02} = P_{cu}$$

Iron loss = Copper loss.

From the above equation, the load current corresponding to maximum efficiency is given by,

$$I_2 = \sqrt{\frac{P_i}{R_{02}}}$$

If we are given iron loss and full load copper loss, then the load corresponding to the maximum efficiency

$$= \text{Full load kVA} \times \sqrt{\frac{\text{Iron Loss}}{\text{Full load Copper loss}}}$$

Condition for maximum efficiency.

output power $= V_2 I_2 \cos\phi_2$.

If R_{02} is the total resistance of the transformer referred to secondary, then, Total copper loss $P_{cu} = I_2^2 R_{02}$.

$$= P_i + P_{cu}$$

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + P_{cu}}$$

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{02}}$$

Dividing both numerator and denominator by I_2,

$$\eta = \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + \frac{P_i}{I_2} + I_2 \cdot R_{02}}$$

The condition for maximum efficiency is obtained by differentiating the denominator and equating it to zero.

$$\frac{d}{dI_2}(\text{denominator}) = 0$$

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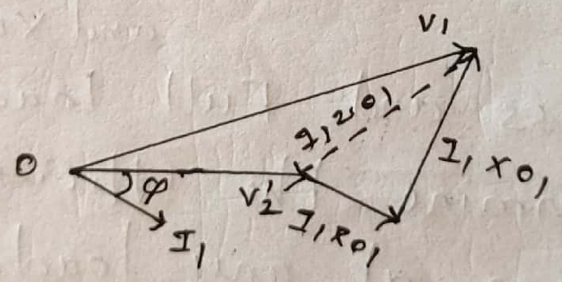
Voltage regulation:

The regulation of a transformer is defined as reduction in magnitude of the terminal voltage due to load, with respect to the no-load terminal voltage.

$$\% \text{ regulation} = \frac{|V_2 \text{ on no load}| - (|V_2 \text{ when loaded}|)}{|V_2 \text{ on no-load}|} \times 100$$

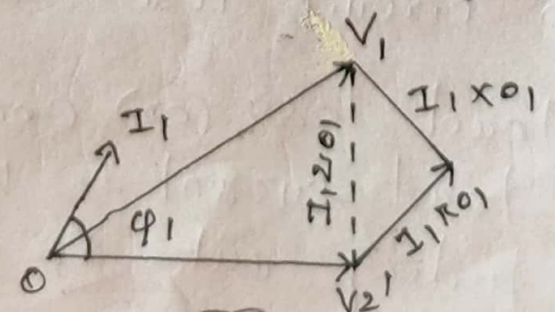
For Lagging power factor

$$\% \text{ Regulation} = \frac{I_1 R_{01} \cos\varphi + I_1 X_{01} \sin\varphi}{V_1} \times 100$$



For Leading power Factor

$$\% \text{ Regulation} = \frac{I_1 R_{01} \cos\varphi - I_1 X_{01} \sin\varphi}{V_1}$$



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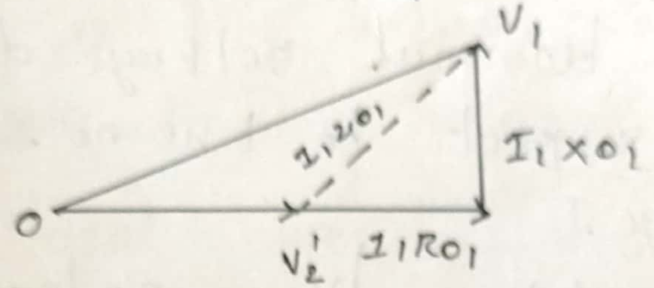
For unity power factor,

$$\% \text{ Regulation} = \frac{I_1 R_{01}}{V_1} \times 100$$



Testing of Transformers.

i. Open circuit test (or) no load test.

ii. Short circuit test (or) Impedance test.

By using these two tests we can find,

1. Circuit constants [$R_0, X_0, R_{01}, X_{01}, R_{02}$ and X_{02})
2. Core loss and Full load copper loss.
3. Predetermine the efficiency and voltage regulation at any load.

Open circuit test:

The open circuit test is useful to find,

i. No load loss (or) core loss

ii. No load current I_0 which is helpful in finding out R_0 and X_0.

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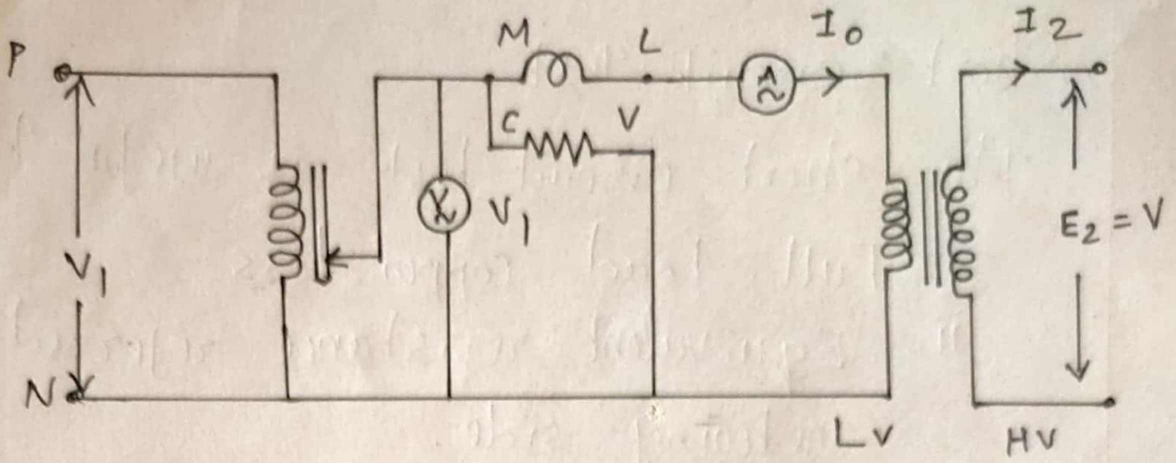

Wattmeter reading $= W_0$

No load current $=$ Ammeter reading $= I_0$

Applied voltage $=$ voltmeter reading $= V_1$

Input power $W_0 = V_1 I_0 \cos\phi_0$

No load power factor $\cos\phi_0 = \dfrac{W_0}{V_1 I_0}$

$$\phi_0 = \cos^{-1}\left[\frac{W_0}{V_1 I_0}\right]$$

No load wattful component $I_W = I_0 \cos\phi_0 = \dfrac{W_0}{V_1}$

No load magnetising component $I_\mu = I_0 \sin\phi_0 = \sqrt{I_0^2 - I_W^2}$

No load resistance $R_0 = \dfrac{V_1}{I_W} = \dfrac{V_1^2}{W_0}$

No load reactance $X_0 = \dfrac{V_1}{I_\mu} = \dfrac{V_1}{\sqrt{I_0^2 - I_W^2}}$

Thus open circuit test gives no load loss P_i, I_W, I_μ, R_0 and X_0.

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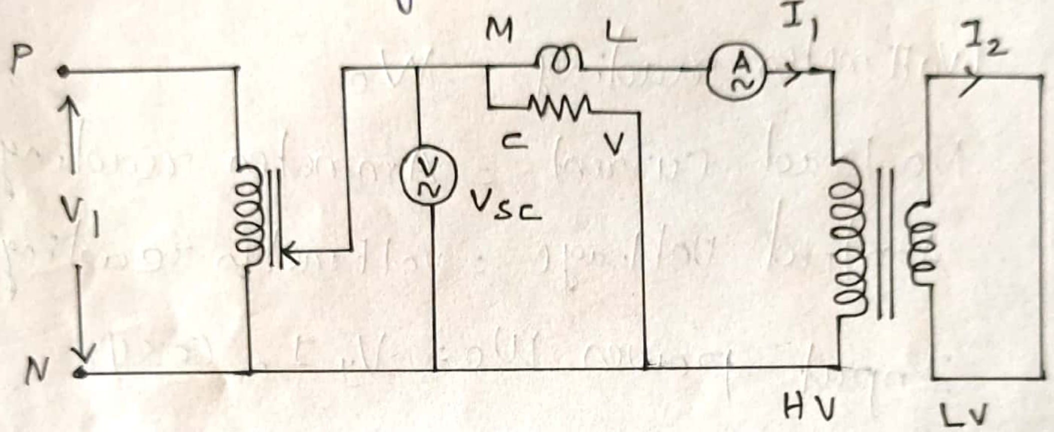
Short circuit Test:

The short circuit test is useful to find,

i. Full load copper loss

ii. Equivalent resistance referred to metering side.



Full load Cu loss P_{cu} = wattmeter reading = W_s

Applied voltage = voltmeter reading = V_{sc}

Full load primary current = Ammeter reading = I_1

$$P_{cu} = I_1^2 R_1 + I_1^2 R_2' = I_1^2 R_{01}$$

$$R_{01} = \frac{P_{cu}}{I_1^2}$$

Total impedance referred to primary

$$Z_{01} = \frac{V_{sc}}{I_1}$$

Total leakage reactance referred to

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Primary $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$

Short circuit power factor $\cos\varphi_s = \frac{P_{cu}}{V_{sc} I_1}$

Thus short circuit test gives full load Cu loss, R_{01}, X_{01} and $\cos\varphi_s$.

From the above two test

Efficiency for (n) load $= \frac{[n \times \text{Full load KVA}] \times p.f}{\left[n \times \text{Full load KVA} \times p.f\right] + P_i + n^2 P_{cu}}$

Ex: Obtain the approximate equivalent circuit of a given 200/2000 V, single phase 25 KVA transformer having the following test results.

O.C test: 200V, 6A, 350W on L.V side

S.C test: 70V, 15A, 600W on H.V side.

Solution:

O.C Test

Primary voltage $V_1 = 200V$

No load input current $I_0 = 6A$

No load input power $P_0 = 350W$.

$P_0 = V_1 I_0 \cos\varphi_0$

No load i/p p.f $\cos\varphi_0 = \frac{P_0}{V_1 I_0}$

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$$= \frac{350}{200 \times 6}$$

$$\cos\phi_0 = 0.2916$$

$$\sin\phi_0 = 0.956$$

Wattfull component $I_W = I_0 \cos\phi_0$

$$= 6 \times 0.2916 = 1.75 \text{ A}$$

Resistance representing the core loss

$$R_0 = \frac{V_1}{I_W} = \frac{200}{1.75} = 114.28\,\Omega$$

Wattless component $I_\mu = I_0 \sin\phi_0$

$$= 6 \times 0.956 = 5.736 \text{ A.}$$

Magnetising reactance $X_0 = \frac{V_1}{I_\mu} = \frac{200}{5.736} = 34.86\,\Omega$

S.C Test:

Short ckt voltage $V_{sc} = 70\text{V}$

Short ckt current $I_{sc} = 15\text{A}$

Full load copper loss $W_{sc} = 600\text{ W}$

Impedance of transformer referred to h.v side

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{70}{15} = 4.66\,\Omega$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{600}{15^2} = 2.66\,\Omega$$

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Transformation ratio $K = \frac{V_2}{V_1} = \frac{2000}{200} = 10$

Referred to 200V side

$$Z_{01} = \frac{Z_{02}}{K^2} = \frac{4.66}{10^2} = 0.0466\,\Omega$$

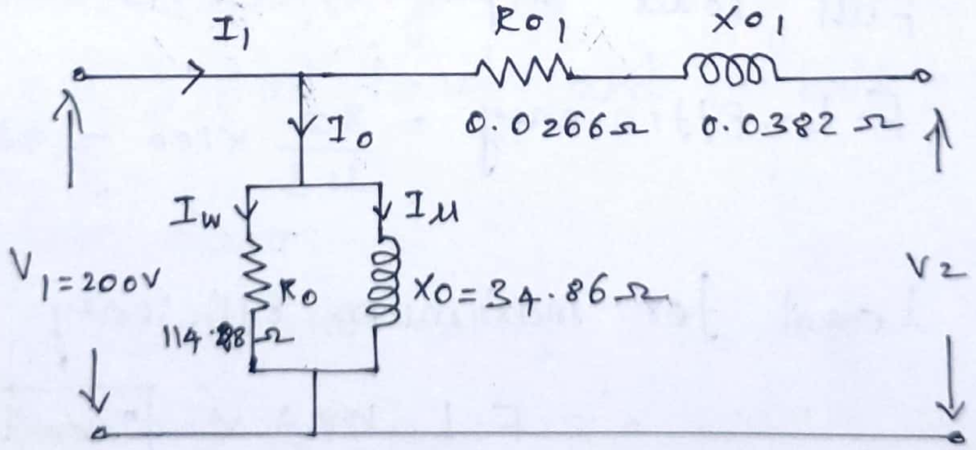
$$R_{01} = \frac{R_{02}}{K^2} = \frac{2.66}{10^2} = 0.0266\,\Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

$$= \sqrt{0.0466^2 - 0.0266^2}$$

$$= 0.0382\,\Omega$$

Approximate equivalent circuit.



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Ex: A 40 KVA transformer has iron loss of 450W and full load copper loss of 850W. If the power factor of the load is 0.8 lagging. calculate (i) Full load efficiency (ii) the load at which maximum efficiency occurs and (iii) the maximum efficiency.

Solution :-

i. Total full load losses = 450 + 850
= 1300 W
= 1.3 KW

Full load output = 40 × 0.8 = 32 KW

Full load input = 32 + 1.3 = 33.3 KW

$$\text{F.L Efficiency} = \frac{32}{33.3} \times 100 = 96.1\%$$

ii. Load for maximum efficiency

$$= \text{F.L. KVA} \times \sqrt{\frac{\text{Iron Loss}}{\text{F.L. Cu Loss}}}$$

$$= 40 \times \sqrt{\frac{450}{850}} = 29.1 \text{ KVA.}$$

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iii) For maximum efficiency

Iron Loss = Copper loss

Total loss = 450 + 450 = 900 W

output power = 29.1 × 0.8 = 23.3 kW

Input power = 23.3 + 0.9 = 24.2 kW.

$$\text{Maximum efficiency} = \frac{23.3}{24.2} \times 100$$

$$= 96.28\%$$

Ex: In a 25 kVA, 2000/200 V, 1φ transformer, the iron and Full load copper losses are 350 and 400 W respectively. calculate the efficiency at unity power factor on (i) Full load and (ii) half full load.

Given data:

Transformer rating = 25 kVA

Primary voltage = 2000 V

secondary voltage = 200 V

Iron loss P_i = 350 W

Full load Cu loss = 400 W.

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Solution:

(i) $n = 1$, $\cos\varphi = 1$

$$\eta = \frac{n\,\text{KVA}\cdot\cos\varphi}{n\,\text{KVA}\cos\varphi + P_i + n^2 P_{CuFL}}$$

$$= \frac{1 \times 25 \times 10^3 \times 1}{(1 \times 25 \times 10^3 \times 1) + 350 + 1^2 \times 400} \times 100$$

$$= \frac{25000}{25000 + 350 + 400} \times 100$$

$$\boxed{\eta = 97.08\,\%}$$

(ii) $n = \frac{1}{2}$, $\cos\varphi = 1$

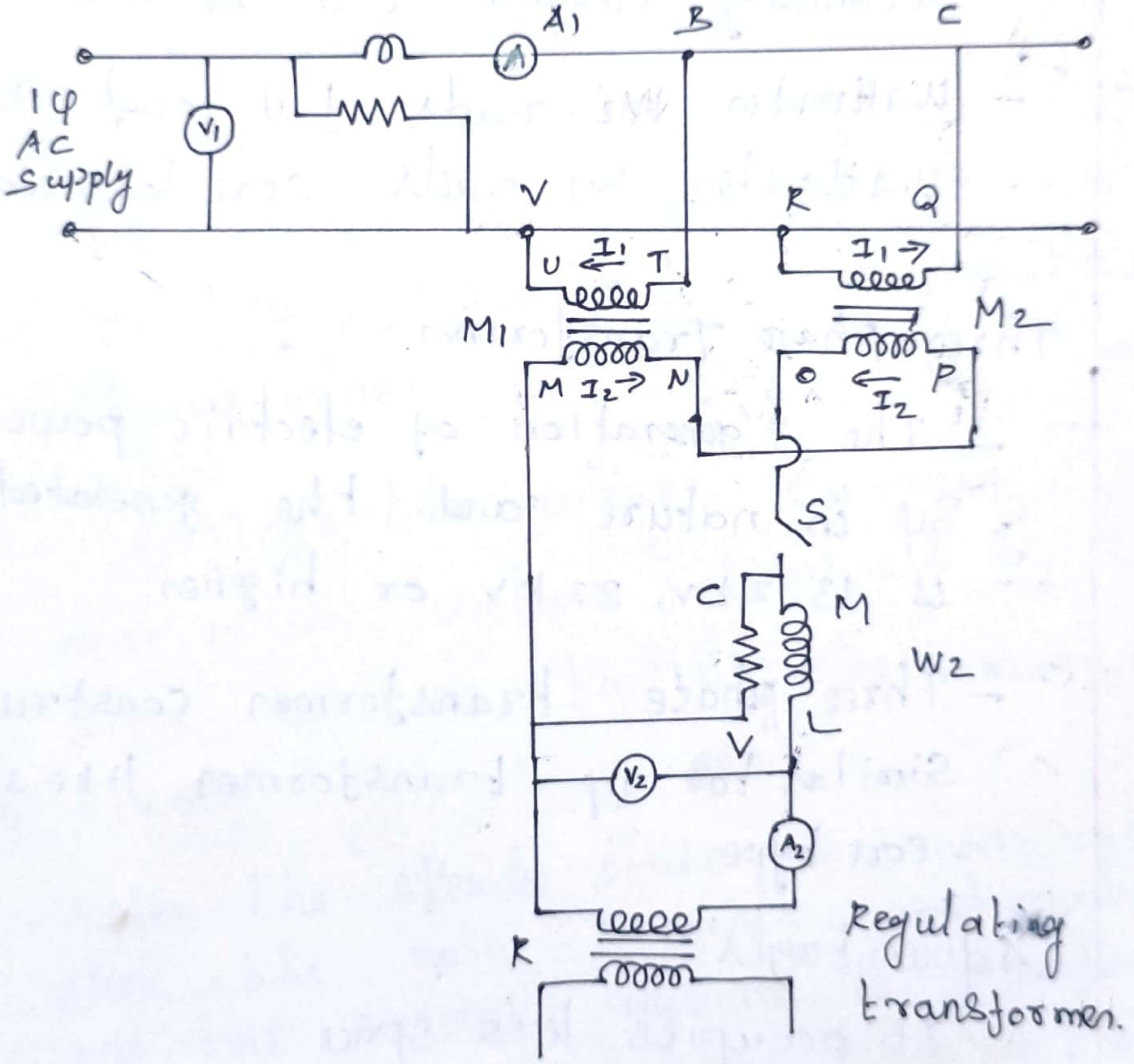
$$\eta = \frac{\frac{1}{2} \times 25 \times 10^3 \times 1}{\left(\frac{1}{2} \times 25 \times 10^3 \times 1\right) + 350 + \left(\frac{1}{2}\right)^2 \times 400} \times 100$$

$$= \frac{12500}{12500 + 350 + 100} \times 100$$

$$\boxed{\eta = 96.52\,\%}$$

Sumpner's test or back to back test:

- This test requires 2 similar transformer.
- These two transformers are fully loaded and the power taken from the supply is that necessary for supplying the iron and copper losses of both transformers.



- The primaries of the two transformers M_1, M_2 are connected in parallel across the AC supply.

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- with switch S open, the wattmeter W_1 reads only core loss of the two transformers because the transformers are under no load.

- By proper variation of R, full load secondary current can be made to flow.
- Wattmeter W_2 reads full load copper loss
- wattmeter W_1 reads core loss contineously.

Three phase Transformer.

- The generation of electric power is 3φ in nature and the generated voltage is 13.2 kV, 22 kV or higher.

- Three phase transformer construction is similar to 1φ transformer like shell or core type.

Advantages.

1. It occupies less space
2. It has less weight
3. Low cost
4. Easy to handle
5. Easy transportation.

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Inrush current:

- The transformer inrush current is the maximum instantaneous current drawn by the primary of the transformer when their secondary is open circuit.

- The inrush current does not create any permanent fault, but it causes an unwanted switching in the circuit breaker of the transformer.

- During the inrush current, the maximum value obtained by the flux is over twice the normal flux.

- After the steady state maximum value of flux, the core becomes saturated and the current required to produce the rest of flux is very high. so the transformer primary will draw a very high peak current from the source. This is known as the transformer inrush current or magnetising inrush current.

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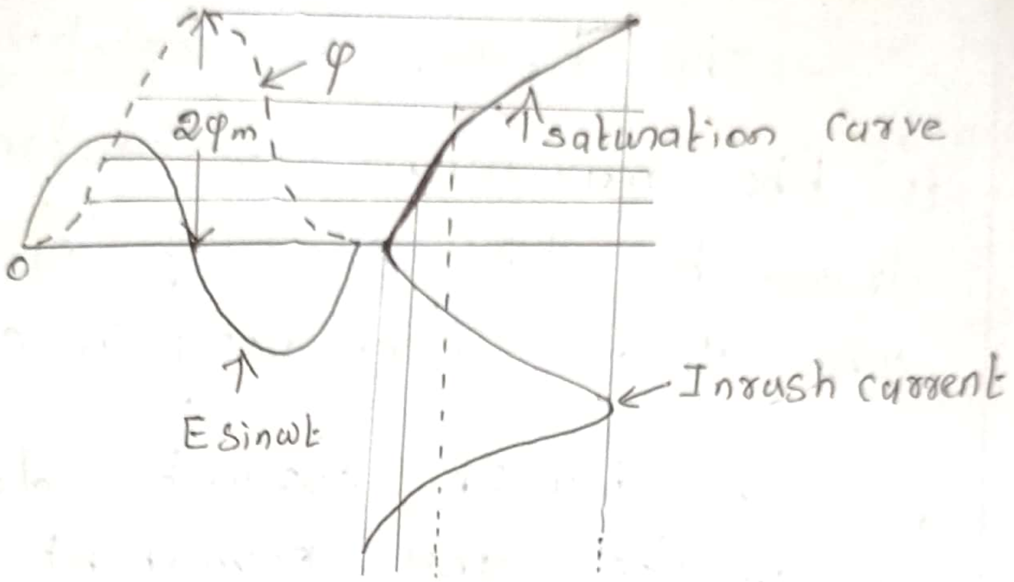

This current is transient in nature and exist for few milliseconds. The inrush current may be up to 10 times higher than normal rated current of transformer.

It does not create any permanent fault in transformer, but still inrush current in power transformer is a problem, because it interferes with the operation of circuits as they have been designed to function.

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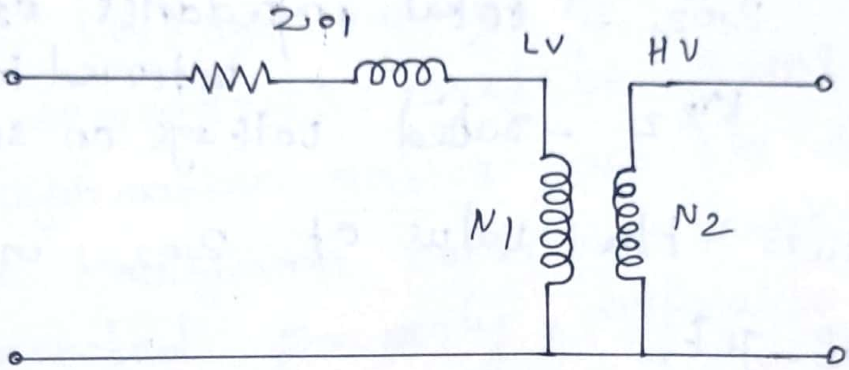
Per Unit Representation:

The approximate equivalent circuit of a two winding transformer with all impedances referred to primary (low-voltage) side is shown in fig.



$$Z_{01} \text{ p.u} = \frac{Z_{01} \times KVA_B}{(KV_{B1})^2 \times 1{,}000} \quad \text{--- (1)}$$

where

Z_{01} – Total impedance of the transformer
KVA_B – Rated KVA of the transformer.
KV_{B1} – Rated voltage of the transformer.

Let us consider the transformer with all its impedances referred to secondary side.

The total impedance of the transformer referred to secondary.

$$Z_{02} = Z_{01} \times \left(\frac{N_2}{N_1}\right)^2 = Z_{01} \frac{(KV_{B2})^2}{(KV_{B1})^2} \quad \text{--- (2)}$$

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The perunit impedance on secondary side

$$Z_{02}\ p.u = \frac{Z_{02} \times KVA_B}{(KV_{B2})^2 \times 1000} \quad \text{--- (3)}$$

where

Z_{02} – total impedance of the transformer referred to secondary.

KV_{B2} – rated voltage on secondary side.

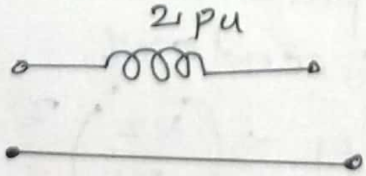
sub the value of Z_{02} in eqn (3)

we get,

$$Z_{02}\ p.u = \frac{Z_{01} \times (KV_{B2})^2}{(KV_{B1})^2} \times \frac{KVA_B}{(KV_{B2})^2 \times 1000}$$

$$\boxed{Z_{02\,pu} = \frac{Z_{01} \times KVA_B}{(KV_{B1})^2 \times 1000}} \quad \text{--- (4)}$$

Comparing the above equns. we conclude that the perunit impedance is the same regardless of the side from which it is viewed.



$\therefore$ the p.u impedance $Z_{01}\ pu = Z_{02}\ pu = Z\ p.u.$

Parallel operation of single phase Transformers.

When two transformers are connected in parallel, the following conditions should be satisfied.

1. The voltage ratings of both primaries and secondaries must be same i.e, transformation ratios are the same.
2. The transformer polarities must be connected properly, otherwise dead short circuit occurs.
3. The ratio of the equivalent resistance to equivalent reactance of the transformers should be equal.
4. The equivalent impedances should be inversely proportional to the respective kVA ratings of the transformers.

Load shared by transformer A,

$$P_A = V_2 I_A = V_2 . I \frac{Z_B}{Z_A + Z_B} \text{ kVA}$$

Load shared by transformer B,

$$P_B = V_2 I_B = V_2 . I \frac{Z_A}{Z_A + Z_B}$$

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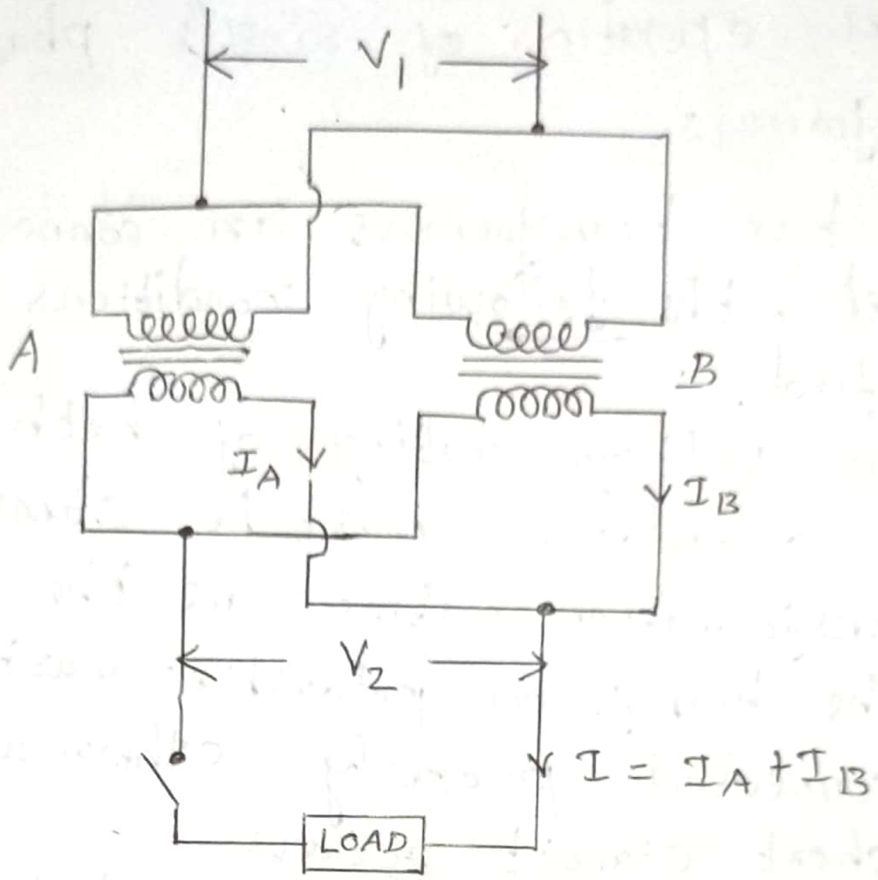

I = Total current $(I_A + I_B)$

The total power $P = P_A + P_B = V_2 \times I$

Hence $$P_A = P \cdot \frac{Z_B}{Z_A + Z_B}$$

$$P_B = P \cdot \frac{Z_A}{Z_A + Z_B}$$

The per unit impedance of two transformers are different, the impedance of both the transformers should be transformed to a common base KVA.

$$Z_A = R_A + j X_A$$
$$Z_B = R_B + j X_B$$

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When two transformers of different KVA ratings,

$$\frac{I_A}{I_B} = \frac{Z_B}{Z_A}$$

$$\frac{I_A}{I_A+I_B} = \frac{Z_B}{Z_A+Z_B}$$

Current through transformer A,

$$I_A = I \times \frac{Z_B}{Z_A+Z_B}$$

Current through transformer B,

$$I_B = I \times \frac{Z_A}{Z_A+Z_B}$$

From this circuit,

$$E_2 = V_2 + I_B . Z_B$$

$$E_1 = V_1 + I_A . Z_A$$

$$I_A Z_A = I_B Z_B = I Z_{AB}$$

$$\frac{I_A}{I_B} = \frac{Z_B}{Z_A}$$

- Assume, no load voltage of both secondaries are same, ie $E_2 = E_A = E_B$
- On no load, no circulating current.

Unequal voltage ratio :-

due to unequal voltage ratio of the transformers, their no-load secondary

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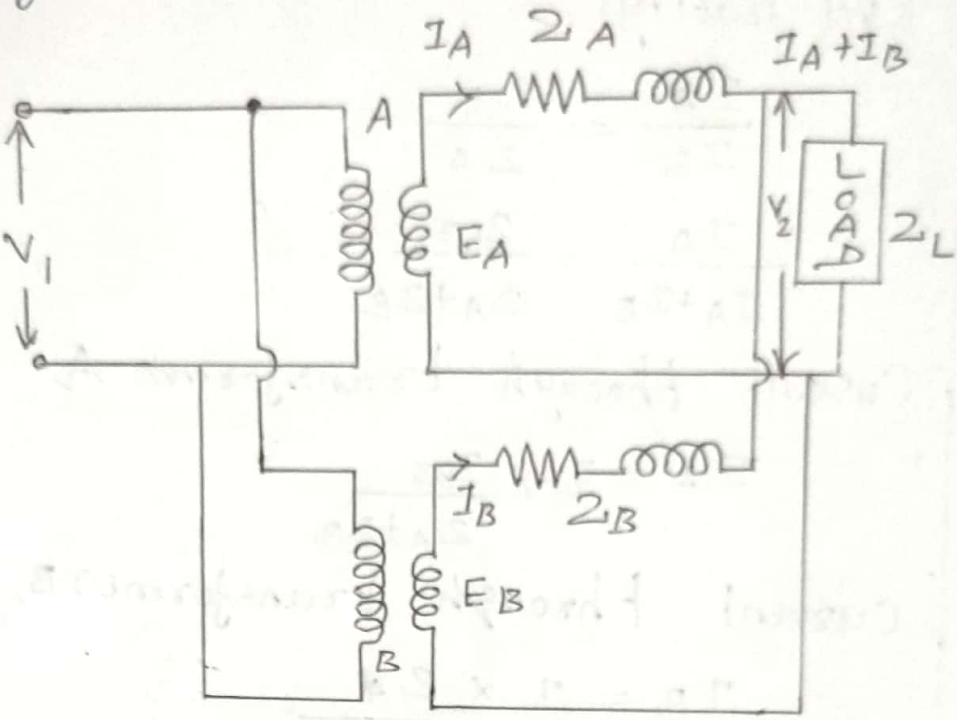
voltages will also be unequal.



Under no load,

$$I_C = \frac{E_A - E_B}{Z_A + Z_B}$$

By applying KVL in the above loop.

$$E_A = V_2 + I_A Z_A$$

$$E_B = V_2 + I_B Z_B$$

but $\quad V_2 = I Z_L = (I_A + I_B) Z_L$

$$E_A = (I_A + I_B) Z_L + I_A Z_A \quad \text{—— (1)}$$

$$E_B = (I_A + I_B) Z_L + I_B Z_B \quad \text{—— (2)}$$

Now $\quad I_B = (I_A + I_B) Z_L + I_A Z_A - (I_A + I_B) Z_L - I_B Z_B$

$$E_A - E_B = I_A Z_A - I_B \cdot Z_B$$

$$(E_A - E_B) + I_B \cdot Z_B = I_A \cdot Z_A$$

$$I_A = \frac{(E_A - E_B) + I_B \cdot Z_B}{Z_A} \quad \text{--- (3)}$$

The above equation, sub in eqn (2), will give,

$$E_B = \left[\frac{(E_A - E_B) + I_A Z_B}{Z_A} + I_B\right] Z_L + I_B Z_B$$

$$I_B = \frac{E_B Z_A - (E_A - E_B) Z_L}{Z_A Z_B + Z_L (Z_A + Z_B)} \quad \text{--- (4)}$$

Eqn (4), sub in eqn (3), we get.

$$I_A = \frac{E_A Z_B + (E_A - E_B) Z_L}{Z_A Z_B + Z_L (Z_A + Z_B)}$$

Load current $I = I_A + I_B$

$$= \frac{E_A Z_B + E_B Z_A}{Z_A Z_B + Z_L (Z_A + Z_B)}$$

Load voltage $V_2 = I Z_L$

$$= \left[\frac{E_A Z_A + E_B Z_A}{Z_A Z_B + Z_L (Z_A + Z_B)}\right] Z_L$$

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Example: Two single phase transformer with equal turns have impedance of $(0.5+j3)\,\Omega$ and $(0.6+j10)\,\Omega$ with respect to the secondary. If they operate in parallel, determine how they will share total load of 100 kW at power factor 0.8 lagging.

Given data,

Impedance of transformer A, $Z_A = (0.5+j3)\,\Omega$

Impedance of transformer B, $Z_B = (0.6+j10)\,\Omega$

Total load to be shared $= 100\,kW$ at $\cos\phi = 0.8$ lagging

To Find :

Load shared by transformer A & B.

Soln:- Total load in KVA $= \dfrac{kW}{\cos\phi} = \dfrac{100}{0.8}$

$$= 125\ kVA.$$

Hence, load to be shared $P = 125 \angle -36.9^\circ$ kVA.

$$Z_A = (0.5+j3)\,\Omega = 3.04 \angle 80.53^\circ\,\Omega$$

$$Z_B = (0.6+j10)\,\Omega = 10 \angle 86.56^\circ\,\Omega$$

$$Z_A + Z_B = 0.5+j3 + 0.6+j10$$

$$= (1.1+j13)\,\Omega$$

$$= 13.04 \angle 85.16^\circ\,\Omega$$

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Load shared by transformer A, $P_A = P \times \dfrac{Z_B}{Z_A + Z_B}$

$$= \frac{125 \angle -36.9^\circ \times 10 \angle 86.56^\circ}{13.04 \angle 85.16^\circ}$$

$$= 125 \angle -36.9^\circ \times 0.766 \angle 1.4^\circ$$

$$= 95.75 \angle -35.5^\circ \text{ kVA.}$$

Load power of transformer A in terms of kW $= 95.75 \times \cos(-35.5^\circ)$

$$\boxed{P_A = 77.95 \text{ kW}}$$

Load shared by transformer B, $P_B = P \times \dfrac{Z_A}{Z_A + Z_B}$

$$= \frac{125 \angle -36.9^\circ \times 3.04 \angle 80.53^\circ}{13.04 \angle 85.16^\circ}$$

$$= 125 \angle -36.9^\circ \times 0.233 \angle -4.63^\circ$$

$$= 29.125 \angle -41.53^\circ \text{ kVA.}$$

Load power of transformer B in terms of kW $= 29.125 \times \cos(-41.5^\circ)$

$$\boxed{P_B = 21.803 \text{ kW}}$$

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Ex: Two transformers A and B are connected parallel to a load of $(2+j1.5)\,\Omega$. Their impedances in terms of secondary are $Z_A = (0.15+j0.5)\,\Omega$ and $Z_B = (0.1+j0.6)\,\Omega$. Their no load terminal voltage are $E_A = (207+j0)\,V$ and $E_B = (205+j0)\,V$. Find the power o/p and p.f of each transformer.

Soln:-

Using the equation $I_A = \dfrac{E_A Z_B + (E_A - E_B) Z_L}{Z_A Z_B + Z_L(Z_A + Z_B)}$

$$Z_A = (0.15+j0.5)\,\Omega$$
$$Z_B = (0.1+j0.6)\,\Omega$$
$$Z_L = 2+j1.5\,\Omega$$

$$I_A = \frac{207(0.1+j0.6)+(207-205)(2+j1.5)}{(0.15+j0.5)(0.1+j0.6)+(2+j1.5)(0.25+j1.1)}$$

$$= \frac{24.7+j127.2}{-1.435+j2.715} = \frac{129.7\angle 79^\circ}{3.07\angle 117.9^\circ}$$

$$= 42.26\angle -38.9^\circ\,A = (32.89-j26.55)\,A$$

$$I_B = \frac{E_B Z_A - (E_A - E_B) Z_L}{Z_A Z_B + Z_L(Z_A + Z_B)}$$

$$= \frac{205(0.15+j0.5) - 2(2+j1.5)}{-1.435+j2.715}$$

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$$= \frac{103 \angle 75^\circ}{3.07 \angle 117.9^\circ} = 33.56 \angle -42.9^\circ \text{ A}$$

$$= 24.58 - j\,22.84 \text{ A}$$

Now $V_2' = I Z_L = (I_A + I_B) Z_L$

$$= (57.47 - j\,49.39)(2 + j\,1.5)$$

$$= 189 - j\,12.58 = 189.4 \angle -3.9^\circ \text{ V.}$$

power factor angle of transformer A

$$= -3.9^\circ - (-38.9) = 35^\circ$$

$$\cos 35^\circ = 0.818 \text{ (lag)}$$

Power factor angle of transformer B,

$$= \cos[-3.9^\circ - (-42.9^\circ)]$$

$$= 0.776 \text{ (lag)}$$

o/p power of transformer A, P_A

$$= 189.4 \times 42.26 \times 0.818$$

$$\boxed{P_A = 6548 \text{ Watts}}$$

o/p power of transformer B, P_B

$$= 189.4 \times 33.56 \times 0.776$$

$$\boxed{P_B = 4900 \text{ watts.}}$$

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The parameters of approximate equivalent circuit of 4KVA, 200/400 V, 50Hz, 1φ transformer are: $R_p' = 0.15\ \Omega$, $X_p' = 0.37\ \Omega$, $R_o = 600\ \Omega$, $X_m = 300\ \Omega$; when rated voltage of 200V is applied to the primary, a current of 10 A at lagging power factor of 0.8 flows in the secondary winding. Calculate (i) the current in the primary (ii) terminal voltage at the secondary side.

(April / May - 2017)

Soln:- $R_p' = R_{01} = 0.15\ \Omega$

$X_p' = X_{01} = 0.37\ \Omega$

$R_o = 600\ \Omega$

$X_m = 300\ \Omega$

$V_1 = 200\ V$

$I_2' = 10\ A$, p.f $= 0.8$

$R_o = \frac{V_1}{I_W}$, $X_m = \frac{V_1}{I_\mu}$ $\qquad K = \frac{V_2}{V_1} = \frac{400}{200} = 2$

$I_W = \frac{V_1}{R_o}$, $I_\mu = \frac{V_1}{X_m}$ $\qquad \boxed{K = 2}$

$= \frac{200}{600}$, $I_\mu = \frac{200}{800}$

$\boxed{I_W = 0.333\ A,\ I_\mu = 0.666\ A}$

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Transformation ratio $k = \frac{V_2}{V_1} = \frac{220}{1100} = 0.2$

parameters referred to 1100V (H.V) side,

$$Z_{01} = \frac{Z_{02}}{k^2} = \frac{0.125}{0.2^2} = 3.125\ \Omega$$

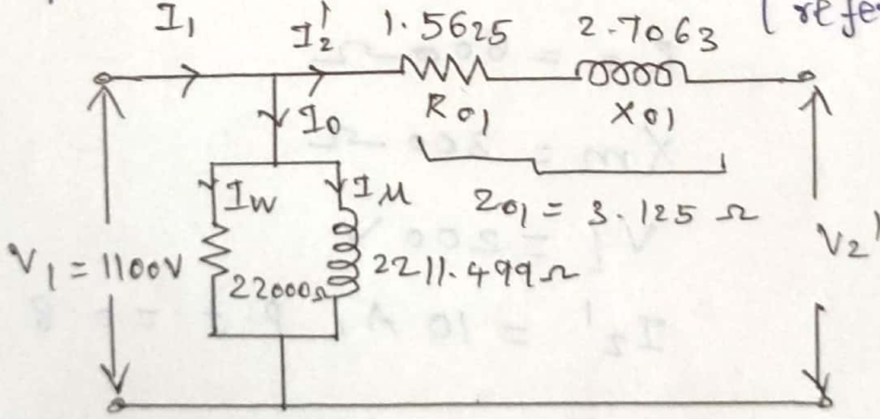
$$R_{01} = \frac{R_{02}}{k^2} = \frac{0.0625}{0.2^2} = 1.5625\ \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

$$= \sqrt{3.125^2 - 1.5625^2}$$

$$\boxed{X_{01} = 2.7063\ \Omega}$$

Equivalent circuit is shown below (referred to primary)



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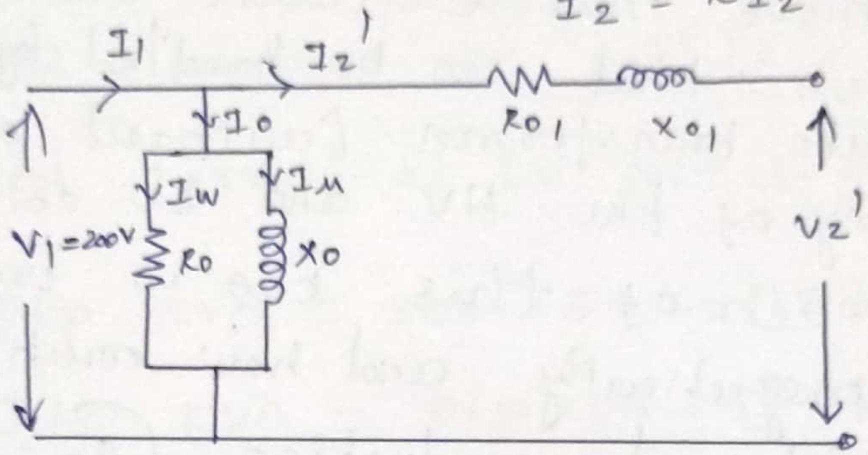
$$I_0 = \sqrt{I_w^2 + I_\mu^2}$$

$$= \sqrt{0.333^2 + 0.666^2}$$

$$I_0 = 0.745 \text{ A}$$

(i) The current in the primary

$$I_2' = K I_2 = 2 \times 10 = 20$$



$$I_1 = \sqrt{I_0^2 + I_2'^2}$$

$$= \sqrt{0.745^2 + 20^2}$$

$$\boxed{I_1 = 20.01387 \text{ A}}$$

(ii) Terminal voltage at the secondary side.

$$V_2' = \frac{V_2}{K}$$

$$= \frac{400}{2}$$

$$\boxed{V_2' = 200 \text{ V}}$$

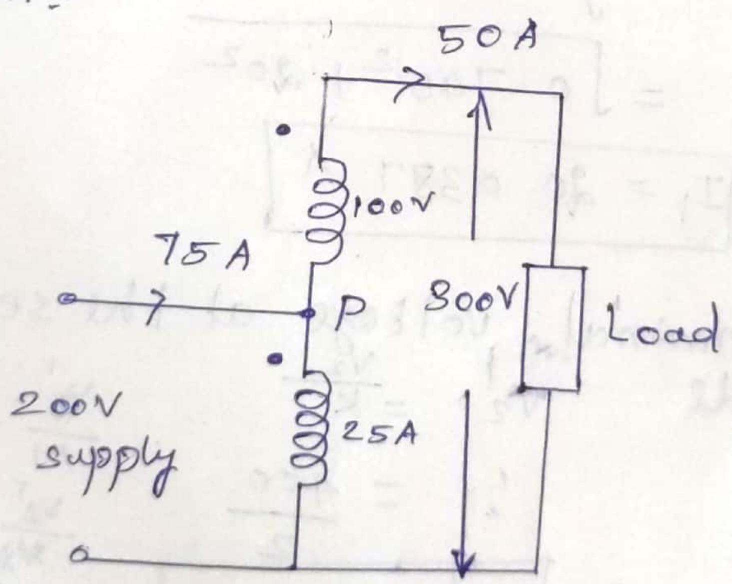
$$\frac{V_2'}{N_1} = \frac{V_2}{N_2}$$

$$\frac{V_2'}{V_2} = \frac{N_1}{N_2}$$

$$V_2' = \frac{V_2}{K}$$

A 5KVA, 200V / 100V, 50Hz, 1φ ideal two winding transformer is used to step up a voltage of 200V to 300V by connecting it like an auto transformer. Show the connection diagram to achieve this. Calculate the maximum KVA that can be handled by the auto transformer (without over loading any of the HV and LV coil). How much of this KVA is transferred magnetically and how much is transfered by electrical conduction. (April/May 2014)

Soln:-



Rated voltage of High voltage coil

$= 200V$

Rated voltage of low voltage coil

$= 100V$

$$K = \frac{200}{100} = 2$$

Rated current of HV coil is $= \frac{5000}{200} = 25A$

Rated current of LV coil is $= \frac{5000}{100} = 50A$

O/P KVA $= 300 \times 50 = 15 KVA$

I/P KVA $=$ O/P KVA $= 15 KVA$

Current drawn from the supply

$$= \frac{15000}{200} = 75A$$

Current through high vg. coil

$= 75 - 50 = 25A$

KVA transfer magnetically = KVA of either HV or LV coil

$= 200 \times 25 = 100 \times 50$

$= 5KVA$

KVA transfered electrically

= total KVA transfered − KVA transfered magnetically

$= 15 - 5$

$$\boxed{kVA_{\text{magnetically}} = 10\,kVA.}$$

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Example The following data were obtained on a 20 kVA 150 Hz, 2000/200 V distribution transformer. Draw the approximate equivalent circuit of transformer referred to the HV and LV sides respectively. Nov/Dec 2016

	voltage (V)	current (I)	power (W)
OC test with HV-open circuited	200	4	120
SC test with LV short circuited	60	10	300.

solution:-

OC test (LV side) $Y_0 = \frac{4}{200} = 2\times10^{-2}\ ℧$

$G_i = \frac{120}{(200)^2} = 0.3\times10^{-2}\ ℧$

$B_m = \sqrt{Y_0^2 - G_i^2}$

$= 1.98\times10^{-2}\ ℧$

SC test (HV side) $Z_1 = \frac{60}{10} = 6\ \Omega$

$R = \frac{300}{10^2} = 3\ \Omega$

$X = \sqrt{Z_1^2 - R^2} = 5.2\ \Omega$

Transformation ratio $\frac{N_H}{N_L} = \frac{2000}{200} = 10$

Equivalent circuit referred to the HV side,

$G_i(HV) = 0.3\times10^{-2} \times \frac{1}{(10)^2} = 0.3\times10^{-4}\ ℧$

$B_m(HV) = 1.98\times10^{-2} \times \frac{1}{(10)^2} = 1.98\times10^{-4}\ ℧$

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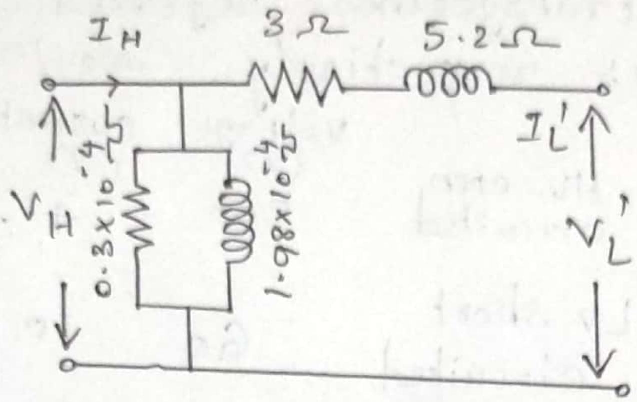
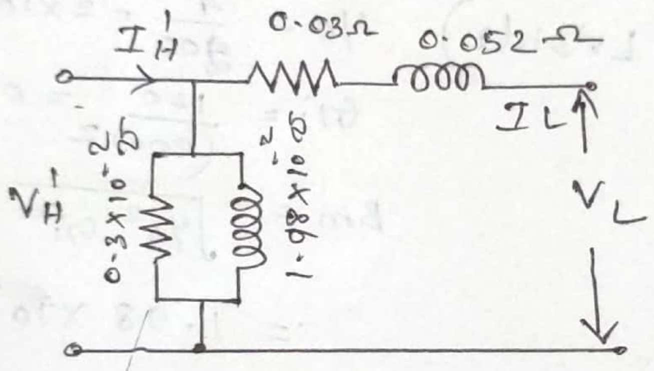
Equivalent circuit referred to the Lv side

$$R(LV) = 3 \times \frac{1}{10^2} = 0.03\,\Omega$$



(a) Referred to HV.



(b) Referred to LV

$$X_{(LV)} = 5.2 \times \frac{1}{10^2}$$

$$\boxed{X_{LV} = 0.052\,\Omega}$$

A transformer on no-load has a core-loss of 50W, draws a current of 2A (rms). and has an induced emf of 230V (rms). Determine the no-load power factor, core loss current and magnetising current. Also calculate the no-load circuit parameters of the transformer. Neglect winding resistance and leakage flux. (NOV/DEC 2018)

soln:-

$$p.f \quad \cos\theta_0 = \frac{50}{2\times230} = 0.108 \text{ lag.}$$

$$\theta_0 = 83.76^\circ$$

Magnetizing current $I_m = I_0 \sin\theta_0$

$$= 2\sin(\cos^{-1} 0.108)$$

$$= 1.988 \text{ A}$$

core loss current $I_i = I_0 \cos\theta_i$

$$= 2 \times 0.108$$

$$= 0.216 \text{ A}$$

In the no-load circuit model core loss is given by,

$$G_i V_1^2 = P_i$$

$$G_i = \frac{P_i}{V_1^2} = \frac{50}{(230)^2} = 0.945\times10^{-3}\ \mho$$

Also $I_m = B_m V_1$

(or) $B_m = \frac{I_m}{V_1}$

$= \frac{1.988}{230}$

$$\boxed{B_m = 8.64 \times 10^{-3} \ \mho}$$

An electromagnetic relay has an exciting coil of 800 turns. The coil has a cross section of 5 cm × 5 cm. Find (a) coil inductance if the air gap length is 0.5 cm. (b) Field energy stored for a coil current of 1.25 A (c) permeance at air gap. (April / May 2018)

soln:-

(i) Permeance at air gap $= \frac{\mu_0 \times 5 \times 5 \times 10^{-4}}{0.5 \times 10^{-2}}$

$= 4\pi \times 10^{-7} \times 10^{-2}$

$= 6.28 \times 10^{-7}$

Coil inductance $= N^2 \times$ permeance

$= 800 \times 800 \times 6.28 \times 10^{-7}$

$= 0.402$ H.

(ii) Energy stored in magnetic field

$= \frac{1}{2} L i^2 = \frac{1}{2} \times 0.402 \times 1.25^2$

$= 0.314$ joule.

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(8)

iii) $W_m = \frac{1}{2} L(x) i^2$

$$= \frac{1}{2}\left[\frac{N^2 \mu_0 A}{x}\right]$$

$$= \frac{\frac{1}{2} \times 800 \times 800 \times 4\pi \times 10^{-7} \times 5 \times 5 \times 10^{-4}}{x} \times i^2$$

$$= \frac{1.005 \times 10^{-3}}{x} \times i^2$$

$$F_f = \frac{\delta}{\delta x}\left[i^2 \times \frac{1.005 \times 10^{-3}}{x}\right]$$

$$= \left[1.005 \times 10^{-3}\right] \times i^2 \times \frac{-1}{x^2}$$

This is to be evaluated at $x = 0.5 \times 10^{-2}$

$$= \frac{-1.005 \times 10^{-3} \times 1.25 \times 1.25}{(0.5 \times 10^{-2})^2}$$

$$= -62.8 \text{ NW}$$

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A 11000/230 V, 150 kVA 1φ 50 Hz transformer has approximately core loss of 1.4 kW and F.L copper loss of 1.6 kW. Determine (i) The kVA load for maximum efficiency and the value of maximum efficiency at unity p.f (ii) The efficiency at 0.8 p.f leading.

(April/May-2018)

soln:-

Load kVA for maximum efficiency is given by = F.L. kVA × $\sqrt{\dfrac{\text{Iron loss}}{\text{F.L Cu loss}}}$

$$= 150 \times \sqrt{\frac{1.4}{1.6}} = 140.31 \text{ kVA.}$$

$$\text{Total loss} = 1.4 + \cancel{1.6}\ 1.4 = 2.8 \text{ kW}$$

$$\cos \phi_2 = 1$$

$$\eta_{max} = \frac{140.31 \times 1}{(140.31 \times 1) + 2.8} \times 100$$

$$\eta_{max} = 98.04\%$$

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(ii) Efficiency at half full load 0.8 p.f leading $n = \frac{1}{2}$

$$\cos\varphi = 0.8$$

$$\eta = \frac{n\,KVA \times \cos\varphi}{n\,KVA \times \cos\varphi + P_i + n^2 P_{cu}}$$

$$= \frac{0.5 \times 150 \times 0.8}{(0.5 \times 150 \times 0.8) + 1.4 + (0.5)^2 \times 1.6} \times 100$$

$$= \frac{60}{60 + 1.8} \times 100$$

$$\boxed{\eta = 97.08\%}$$

A 20 KVA, 2000/200 V, 50 Hz, 1φ transformer has the following parameters. $r_1 = 2.8\,\Omega$, $r_2 = 0.02\,\Omega$, $x_1 = 4.2\,\Omega$ and $x_2 = 0.6\,\Omega$

calculate :-

1) Equivalent resistance, leakage reactance and impedance referred to HV side.
2) Equivalent resistance, leakage reactance and impedance referred to LV side.
3) Full load Cu loss.

(April/May - 2018)

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Soln :-

$$k = \frac{V_2}{V_1} = \frac{200}{2000} = 0.1$$

i. Equivalent resistance as referred to primary R_{o1}

$$R_{o1} = R_1 + R_2' = R_1 + \frac{R_2}{k^2}$$

$$R_{o1} = 2.8 + \frac{0.02}{0.1^2} = 4.8\,\Omega$$

ii) Equivalent reactance as referred to primary X_{o1}

$$X_{o1} = X_1 + X_2' = X_1 + \frac{X_2}{k^2}$$

$$= 4.2 + \frac{0.6}{0.1^2}$$

$$X_{o1} = 64.2\,\Omega$$

iii) Equivalent impedance as referred to primary (Z_{o1})

$$Z_{o1} = \sqrt{R_{o1}^2 + X_{o1}^2} = \sqrt{4.8^2 + 64.2^2}$$

$$= 64.379\,\Omega$$

iv) Equivalent resistance as referred to secondary R_{o2}

$$R_{o2} = R_2 + R_1' = R_2 + R_1 k^2$$

$$= 0.02 + 2.8 \times 0.1^2$$

$$R_{o2} = 0.048\,\Omega$$

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Equivalent reactance as referred to secondary

$$X_{o2} = X_2 + X_1'$$

$$= X_2 + X_1 k^2$$

$$= 0.6 + 4.2 \times 0.1^2$$

$$X_{o2} = 0.642\ \Omega$$

$$Z_{o2} = \sqrt{R_{o2}^2 + X_{o2}^2}$$

$$= \sqrt{0.048^2 + 0.642^2}$$

$$\boxed{Z_{o2} = 0.6437\ \Omega}$$

(iii) Full load Cu loss $= I_1^2 R_{o1}$

$$I_1 = K I_2$$

$$= 0.1 \times 100$$

$$I_1 = 10\ A$$

$$I_2 = \frac{20 \times 10^3}{200}$$

$$= 100$$

$$K = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

$$I_1 = K I_2$$

$$P_{cu} = 10^2 \times 4.8$$

$$P_{cu} = 100 \times 4.8$$

$$\boxed{P_{cu} = 480\ \text{Watt}}$$

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Draw the equivalent circuit of a 1ϕ 1100/220 V transformer on which the following results were obtained.

(i) 1100V, 0.5A, 55W on primary side, secondary being open circuited.

(ii) 10V, 80A, 400W on LV side, high voltage side being short circuited.

Calculate the voltage regulation and efficiency for the above transformer when supplying 100 A at 0.8 p.f lagging.

(April/May 2017)

Gn data:

$$V_1 = 1100V, \quad V_2 = 220V$$

soln:-

OC Test :-

$$V_1 = 1100V, \quad I_0 = 0.5 A$$

No load i/p power $P_0 = 55 W$

$$P_0 = V_1 I_0 \cos\varphi_0$$

$$\cos\varphi_0 = 0.1$$

$$\sin\varphi_0 = 0.9949$$

Wattful component $I_W = I_0 \cos\varphi_0$

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$= 0.5 \times 0.1 = 0.05\,A$

Wattless component

$I_m = I_0 \sin\varphi_0 = 0.5 \times 0.1 = 0.497A$

Resistance representing the core loss

$$R_0 = \frac{V_1}{I_W} = \frac{1100}{0.05} = 22000\,\Omega$$

Magnetizing reactance

$$X_0 = \frac{V_1}{I_m} = \frac{1100}{0.4974} = 2211.499\,\Omega$$

S.C Test :-

Short circuit voltage $V_{sc} = 10V$

short circuit current $I_{sc} = 80\,A$

Losses $W_{sc} = 400W$

Impedance of transformer referred to secondary. $Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{10}{80} = 0.125\,\Omega$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{400}{80^2} = 0.0625\,\Omega$$

$$X_{02} = \sqrt{Z_{02}^2 - R_{02}^2}$$

$$= \sqrt{0.125^2 - 0.0625^2}$$

$$X_{02} = 0.1082\,\Omega$$

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with neat sketch

Ex The O.C and Short circuit test data are given below for a single phase, 5KVA 200V / 400V, 50 Hz transformer.

O.C test from LV side : 200V, 1.25A, 150W.
S.C test from HV side : 20V, 12.5A, 175W.

Draw the equivalent circuit of the transformer (i) referred to low voltage side and (ii) referred to HV side inserting all the parameter values. (April / May 2019)

Soln:-
O.C Test (on H.V side) - The secondary side is open.

Primary voltage $V_1 = 200V$
No load i/p current $I_0 = 1.25A$
No load i/p power $W_0 = 150W$

$$W_0 = V_1 I_0 \cos\phi_0$$

No load i/p power factor $\cos\phi_0 = \frac{W_0}{V_1 I_0}$

$$= \frac{150}{200 \times 1.25}$$

$$\cos\phi_0 = 0.6$$

$$\phi_0 = \cos^{-1} 0.6$$

$$= 53.130$$

$$\sin 53.130 = 0.8$$

wattfull component (working component)

$$I_W = I_0 \cos\phi_0$$

$$= 1.25 \times 0.6$$

$$\boxed{I_W = 0.75 \text{ A}}$$

Resistance representing the core loss (R_0)

$$R_0 = \frac{V_1}{I_W} = \frac{200}{0.75} = 266.667 \Omega$$

wattless component (magnetizing component)

$$I_\mu = I_0 \sin\phi_0$$

$$= 1.25 \times 0.8$$

$$\boxed{I_\mu = 1 \text{ A}}$$

Magnetising reactance (X_0)

$$X_0 = \frac{V_1}{I_\mu} = \frac{200}{1} = 200 \Omega$$

$$\boxed{X_0 = 200 \Omega}$$

S.c test (LV side shorted)

short circuit voltage $V_{sc} = 20$ V

short circuit current $I_{sc} = 12.5$ A

short circuit power or loss $W_{sc} = 175$ W

Impedance of transformer referred to H.V side

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{20}{12.5} = 1.6 \Omega$$

$$W_{sc} = I_{sc}^2 R_{02} = 12.5^2 \times R_{02}$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{175}{12.5^2} = 1.12 \Omega$$

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Transformation ratio $k = \frac{V_2}{V_1} = \frac{20}{200} = \frac{1}{10}$

Referred to low voltage side

$$Z_{01} = \frac{Z_{02}}{k^2} = \frac{1.6}{(1/10)^2} = 160\ \Omega$$

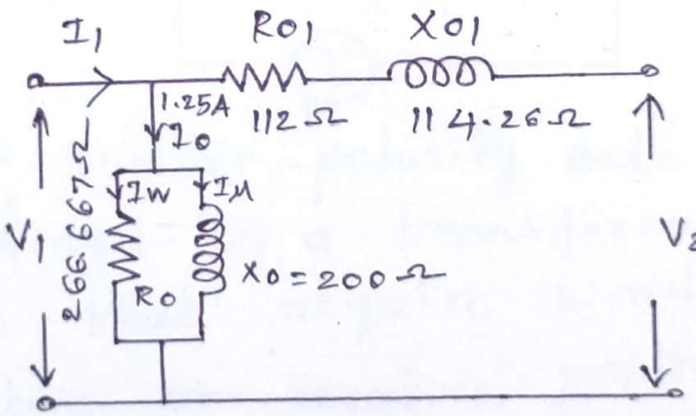
$$R_{01} = \frac{R_{02}}{k^2} = \frac{1.12}{(1/10)^2} = 112\ \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

$$= \sqrt{160^2 - 112^2}$$

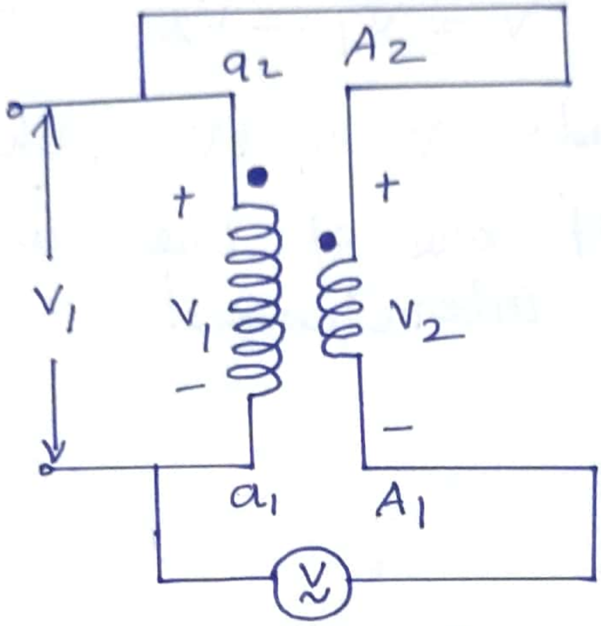
$$\boxed{X_{01} = 114.26\ \Omega}$$

Equivalent circuit referred to the LV side.



POLARITY TEST.

* A polarity test is carried out to find out the terminal having the same instantaneous polarity assuming that the terminals are not marked.



* Similar polarity ends of the two windings of a transformer are those ends that acquire simultaneously positive or negative polarity of emf's induced in them. These are indicated by dot convention.

* usually the ends of the LV winding are labelled with small letter of the alphabet with suffix 1 and 2, while the high voltage winding are labelled by the corresponding capital letter

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with suffix 1 and 2.

* If the polarities of the windings are as marked on the diagram, the voltmeter should read

$$V = V_1 - V_2.$$

If it reads $V = V_1 + V_2$, the polarity marking of one of the windings must be interchanged.

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UNIT V

AUTO TRANSFORMER AND THREE PHASE TRANSFORMER

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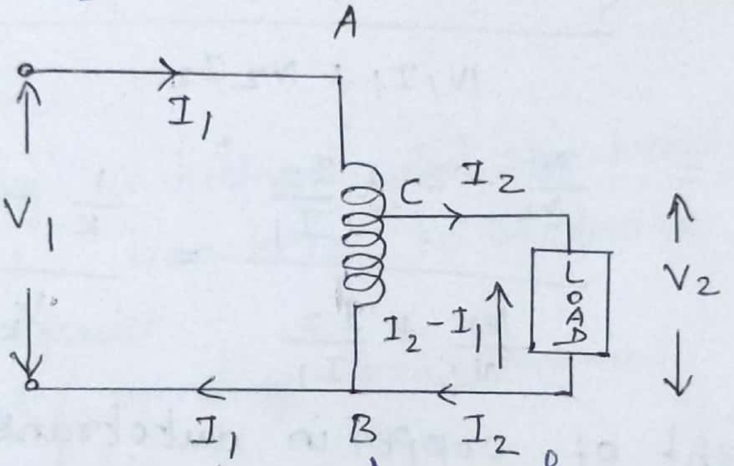
Auto Transformer (or) variac.

A transformer in which part of the winding is common to both the primary and secondary is known as an autotransformer.

The primary is electrically connected to the secondary, as well as magnetically coupled to it.



Fig, AB is primary winding having N_1 turns and BC is secondary winding having N_2 turns. neglecting losses and no load current.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

Saving of copper:-

Weight of copper in section AC $\propto (N_1 - N_2) I_1$

Weight of copper in section BC $\propto N_2 (I_2 - I_1)$

Total weight of copper in autotransformer $\propto (N_1 - N_2) I_1 + N_2 (I_2 - I_1)$

Weight of copper in an ordinary transformer

$$\propto N_1 I_1 + N_2 \cdot I_2$$

$$= \frac{\text{weight of copper in autotransformer } (W_a)}{\text{weight of copper in ordinary transformer } (W_o)}$$

$$= \frac{(N_1 - N_2) I_1 + (I_2 - I_1) N_2}{N_1 I_1 + N_2 I_2}$$

$$= \frac{(N_1 - 2N_2) I_1 + N_2 I_2}{N_1 I_1 + N_2 I_2}$$

$$= \frac{\frac{N_1}{N_2} - 2 + \frac{I_2}{I_1}}{\frac{N_1}{N_2} + \frac{I_2}{I_1}} = \frac{\frac{1}{K} - 2 + \frac{1}{K}}{1/K + 1/K} = 1 - K$$

Weight of copper in autotransformer $= (1-K) \times$ weight of copper in ordinary transformer (W_o)

Saving in copper $= W_o - W_a$

$= W_o - (1-K) W_o$

$= K W_o$

Saving in copper $= K \times$ weight of copper in ordinary transformer.

Advantages:

1. Higher efficiency
2. Small size
3. Smaller exciting current

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4. Lower cost
5. Better voltage regulation
6. Required less copper.

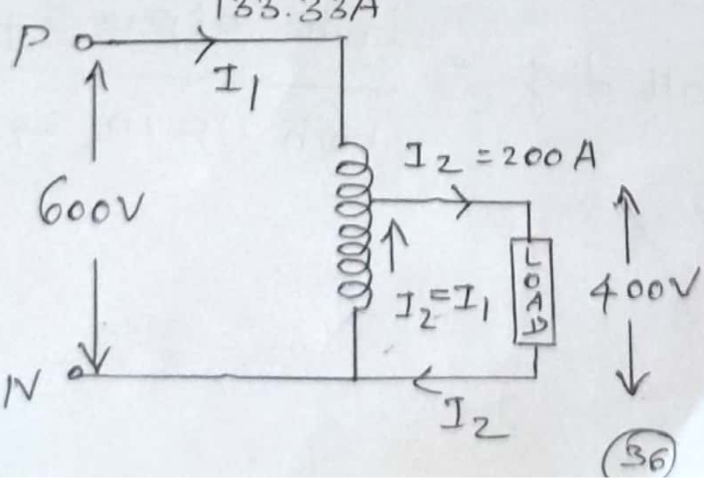
Dis advantages:

1. Direct electrical connection between low tension and high tension sides.
2. High short circuit current.

Applications:-

1. starting induction and synchronous motors.
2. Used as boosters to increase the voltage in AC feeder.
3. Furnace winding transformer.

Ex: The primary and secondary voltages of an auto transformer are 600 V and 400V respectively. show with the aid of a diagram the current distribution in the windings when the secondary current is 200 A. Calculate the economy in copper.



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$$V_1 I_1 = V_2 I_2$$

$$600 \times I_1 = 400 \times 200$$

$$\therefore I_1 = 133.33 A$$

current in the common portion of the winding $= I_2 - I_1$

$$= 200 - 133.33$$

$$= 66.67 \ A$$

saving of copper $= k W_o$

$$= \frac{400}{600} W_o$$

$$= 0.666 \ W_o$$

$\therefore$ % Saving of copper $= 0.666 \times 100$

$$= 66.660 \%$$

All day efficiency:

The ratio of o/p in kwh to i/p in kwh of a transformer over a 24 hour period is known as all-day efficiency.

$$\eta_{all\text{-}day} = \frac{\text{kwh o/p in 24 hours}}{\text{kwh i/p in 24 hours}}$$

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Ex: Find the all day efficiency of 500 kVA distribution transformer whose copper loss and iron loss at full load are 4.5 kW and 3.5 kW respectively. During a day of 24 hours, it is loaded as under.

Number of hours	Loading in kW	powerfactor
6	400	0.8
10	300	0.75
4	100	0.8
4	0	-

(April/May - 2018)

<u>Given</u> :-

Copper loss = 4.5 kW

Iron loss = 3.5 kW

Total o/p in 24 hours = $(500 \times 0.8 \times 6) + (500 \times 0.75 \times 10) + (500 \times 0.8 \times 4)$

$= 2400 + 3750 + 1600$

$= 7750$ kWh

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Iron loss for 24 hours = 3.5×24
$= 84$ kWh

Copper loss for 24 hours

$$= \left(\frac{400/0.8}{500}\right)^2 \times 4.5 \times 6 + \left(\frac{300/0.75}{500}\right)^2 \times 4.5 \times 10 + \left(\frac{100/0.8}{500}\right)^2 \times 4.5 \times 4$$

$= 27 + 28.8 + 1.125$
$= 56.925$ kWh

Total loss = Iron los + copper loss

$= 84 + 56.925$
$= 140.925$ kwh

I/p power = o/p + loss
$= 7750 + 140.925$
$= 7890.925$ kwh

$$\eta_{all\text{-}day} = \frac{\text{O/P Power in kWh}}{\text{I/P Power in kWh}} = \frac{7750}{7890.925} \times 100$$

$$\boxed{\eta_{all\text{-}day} = 98.214\,\%}$$

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Model problem.

Ex: Find all-day efficiency of a transformer having maximum efficiency of 98% at 15 KVA at unity p.f and loaded as follows.

12 hrs — 2 kW at 0.5 p.f lag

6 hrs — 12 kW at 0.8 p.f lag

6 hrs — at no load.

Soln :-

At maximum efficiency

output power = 15 × 1.0 = 15 kW.

$$I/p \text{ power} = \frac{O/p \text{ power}}{\text{efficiency}} = \frac{15}{0.98}$$

$$= 15.306 \text{ kW}$$

Total losses = I/p power − o/p power

$$= 15.306 - 15 = 0.306 \text{ kW}.$$

Full load copper loss = Iron loss = $\frac{\text{Total loss}}{2}$

$$= \frac{0.306}{2} = 0.153 \text{ kW}.$$

Total output $= (2 \times 12) + (12 \times 6) = 96 \text{ kWh}$

Iron loss for 24 hrs $= 0.153 \times 24 = 3.672 \text{ kWh}$

Copper loss for 24 hrs $= \left(\frac{2/0.5}{15}\right)^2 \times 0.153 \times 12 +$

$$\left(\frac{12/0.8}{15}\right)^2 \times 0.153 \times 6$$

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$= 1.04856\ kwh.$

$$\eta_{allday} = \frac{o/p \text{ in } kwh}{I/p \text{ in } kwh} \times 100$$

$$= \frac{96}{96 + 3.672 + 1.04856} \times 100$$

$$\boxed{\eta_{allday} = 95.31\%}$$

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Ex: A 20KVA, 50Hz, ~~2400~~/~~220~~V, 1φ distribution transformer has iron loss of 324W. The copper loss is found to be 100W when delivering half full load current. Determine.

i. Efficiency when delivering full load current at 0.8 lagging p.f and.

ii. The percent of full load when the efficiency will be maximum.

Ex: The maximum efficiency of a single phase 250KVA, 12000/250V transformer occurs at 80% of full load and is equal to 97.5% at 0.8 pf. Determine the efficiency and regulation on full load at 0.8 pf lagging if the impedance of transformer is 9 percent.

Soln:- maximum efficiency occurs at 80% of full load at 0.8 p.f

$$\eta_{max} = 97.5\%$$

$$o/p \text{ power } \eta_{max} = (250 \times 0.8) \times 0.8$$

$$= 160 \text{ kW}$$

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$$I/P \text{ power} = \frac{O/P \text{ Power}}{\eta} = \frac{160}{0.975} = 164.10 kw.$$

$$\therefore \text{Total loss} = 164.10 - 160 = 4.10 kw.$$

Copper loss at 80% of full load

$$= \frac{4.1}{2} = 2.05 kw$$

$$\% R = \frac{\text{Copper loss}}{V_2 I_2} \times 100$$

$$= \frac{2.05}{250} \times 100 = 0.82\% = V_r$$

$$V_x = 9\%$$

$$\% \text{ regulation} = \frac{V_r \cos\phi + V_x \sin\phi}{V_2} \times 100$$

$$= \frac{0.82 \times 0.8 + 9 \times 0.6}{250} \times 200$$

$$\boxed{\% \text{ reg} = 2.42\%}$$

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Ex. Find the all day efficiency of 500 kVA distribution transformer whose copper loss and iron loss at full load are 4.5 kW and 3.5 kW, respectively. During a day of 24 hours, it is loaded as under.

Number of hours	Loading in kW	Power factor
6	400	0.8
10	300	0.75
4	100	0.8
4	0	—

April/May - 2018

An:-

Copper loss = 4.5 kW

Iron loss = 3.5 kW

$$\text{Total o/p in kW} = (6 \times 400) + (10 \times 300) + (4 \times 100)$$

$$= 5800 \text{ kWh}$$

$$\text{Iron Loss for 24 hrs} = 3.5 \times 24 = 84 \text{ kWh}$$

$$\text{Copper loss for 24 hrs} = \left(\frac{\left(\frac{400}{0.8}\right)}{500}\right)^2 \times 4.5 \times 6 +$$

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$$\left(\frac{\left(\frac{300}{0.75}\right)}{500}\right)^2 \times 4.5 \times 10 + \left(\frac{\left(\frac{100}{0.8}\right)}{500}\right)^2 \times 4.5 \times 4$$

$$= 1.6875 \times 10^{12} + 1.8 \times 10^{12} + 7 \times 10^{10}$$

$$= 1.6875 \times 10^{12} + 1.8 \times 10^{12} + 0.07 \times 10^{12}$$

$$= 3.5575 \times 10^{12}$$

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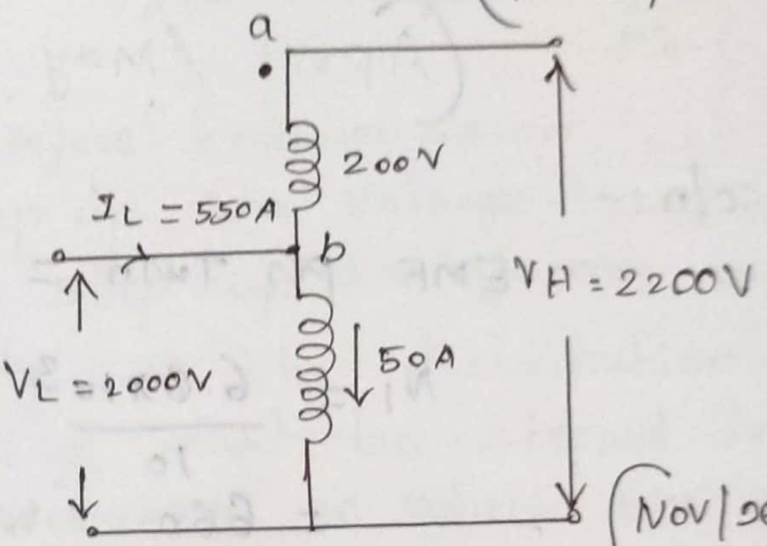
A 1φ, 100 kVA, 2000/200 V two winding transformer is connected as an autotransformer as shown in fig. Such that more than 2000 V is obtained at the secondary. The portion ab is the 200 V winding, and the portion bc is the 2000 V winding. Compute the kVA rating as an atuo transformer. (NOV/DEC 2018)



(Nov/Dec 2018)

Soln:

$$I_1 = I_L - I_2$$

$$I_1 = 550 - 50 = 500 \text{ A}$$

$$\boxed{\begin{array}{l} V_L \times I_L = V_H \times I_1 \\ 2000 \times 550 = 2200 \times 500 \end{array}}$$

volt amp rating $= 2200 \times 500$

$= 1100 \text{ kVA}$

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The emf per turn of a 1φ, 6.6 kV/440V 50 Hz transformer is approximately 10V. Calculate the number of turns in the HV and LV windings and the net cross sectional area of the core for a maximum flux density of core of 1.6T. (April / May - 2018)

soln:-

$$\text{EMF per Turn} = 10V$$

$$N_1 = \frac{6.6 \times 10^3}{10}$$

$$= 660$$

$$N_2 = \frac{440}{10} = 44$$

$$E_1 = 4.44 f_1 N_1 \varphi$$

$$BA = \frac{E_1}{4.44 f \times N_1}$$

$$A = \frac{\cancel{4.44}\ 6.6 \times 10^3}{4.44 \times 50 \times 660 \times 1.6}$$

$$\boxed{A = 0.028\ m^2}$$

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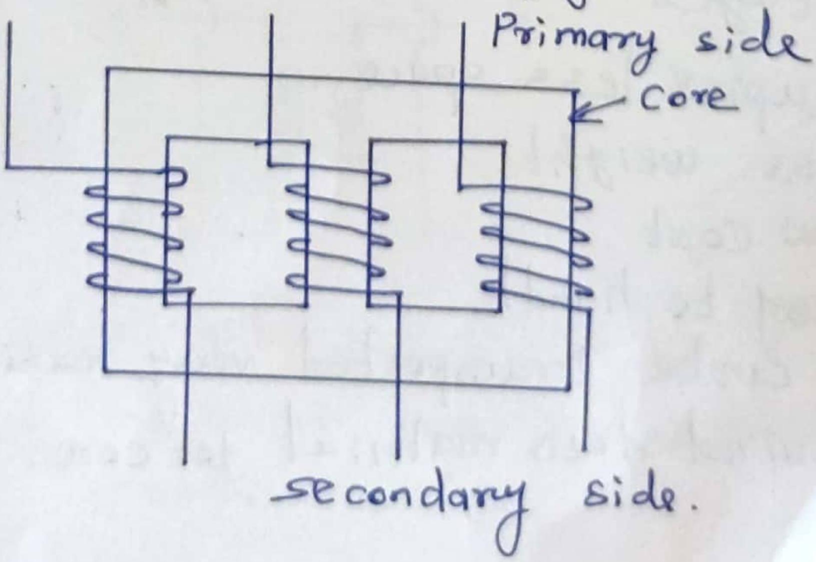
THREE PHASE TRANSFORMERS.

* The generation of electric power is three-phase in nature and the generated voltage is 13.2 KV, 22 KV or higher.

* Transmission of power is carried out at high voltage like 132 KV or 400 KV.

* Before transmission, it is required to step up the voltage and for this a 3 phase step up transformer is required. Similarly, at the distribution sub-station the voltage must be stepped down and it is necessary to reduce the voltage upto 6000 V, 400 V, 230 V and so on.

* Three phase transformer construction is similar to single phase transformer like shell or core type. It is shown in the figure.



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* three phase shell type transformer has three limbs. Here, we use only 'I' core. Around each limb, the primary and secondary windings are placed.

* The operation of 3φ transformer is similar to single phase transformer. Three phase supply is given to the primary winding. Due to this, 3φ flux is produced in the primary winding.

* This flux is linked with secondary winding. Depending upon the number of turns in the secondary, the secondary voltage will be stepped up or stepped down. The primary and secondary windings can be connected in star or delta.

Advantages

1. Occupies less space
2. Less weight
3. Low cost
4. Easy to handle
5. It can be transported very easily.
6. Required less material for core.

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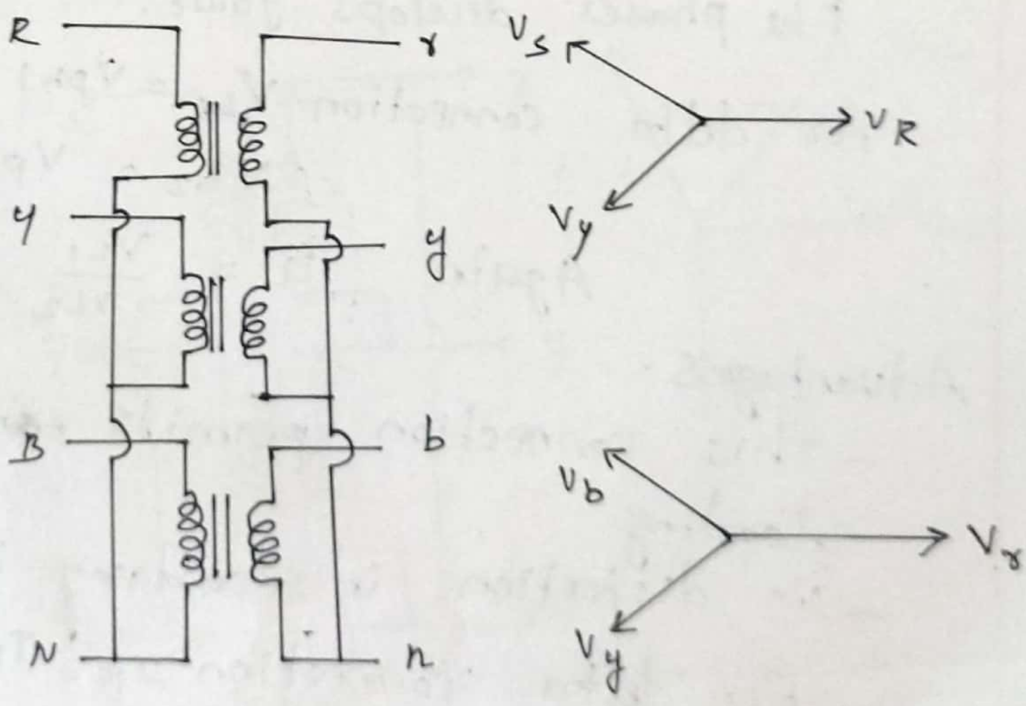
Three phase Transformer Connections.

- Star- star
- Delta- Delta
- Star- Delta
- Delta- star

Star- Star Connection:

This type of connection is most economical for small current rating, high voltage transformer because the phase voltage is $1/\sqrt{3}$ times the line voltage.



Advantages:

- Less no of turns & less quantity of insulation

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- There is no phase shift
- Suitable for 3φ and 4 wire system.

Dis advantages

- Performance is not satisfactory
- Third harmonic present in the alternator voltage.

Delta - Delta connection.

- This arrangement is generally used in system which carry large currents on low voltages and especially when continuity of service must be maintained even though one of the phases develops fault.

For delta connection $V_{L1} = V_{ph1}$ and

$V_{L2} = V_{ph2}$

Again $a = \frac{V_{L1}}{V_{L2}} = \frac{V_{ph1}}{V_{ph2}}$

Advantages:

- This connection permits unbalanced loading.
- No distortion in secondary voltage occurs.
- For delta connection $I_p = I_L/\sqrt{3}$, which makes the connection economical for low voltage transformers.

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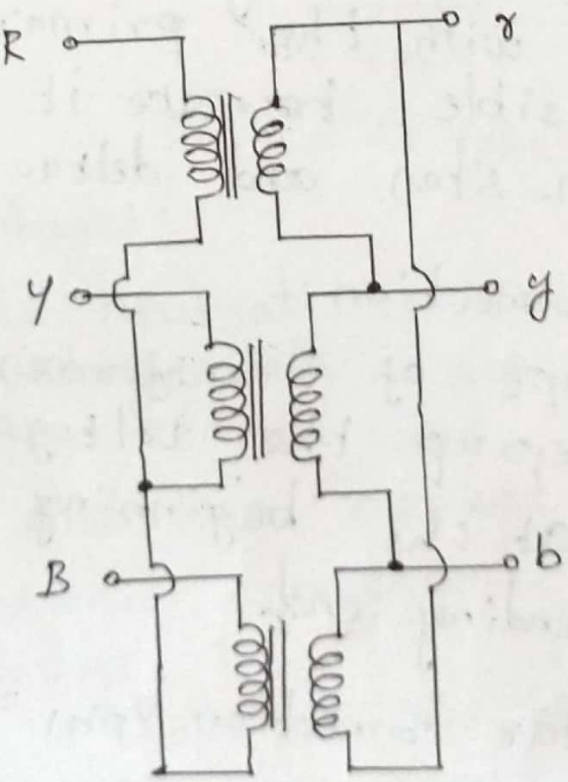
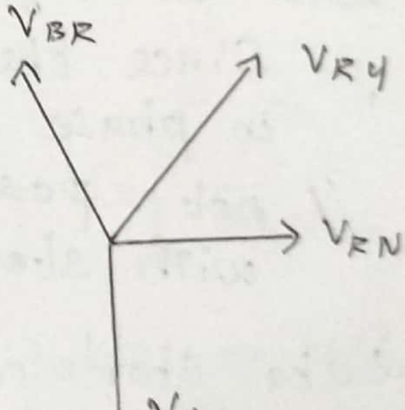
Disadvantages:

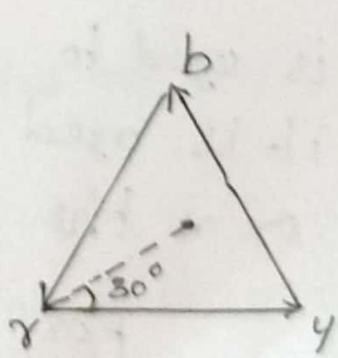
- It is not possible for 3φ four wire system because neutral point is absent.
- This connection is generally used for low voltage transformers.

Star - Delta connection.

This type of connection is used in transformer to step down voltages at the distribution and hence it is used at the receiving side, that is at the receiving side after the transmission.







For star connection $V_{ph1} = \frac{V_{L1}}{\sqrt{3}}$

For delta connection $V_{L2} = V_{ph2}$

$$V_{ph2} = \frac{1}{a} V_{ph1} = \frac{V_{L1}}{a\sqrt{3}}$$

where $a = \frac{V_{ph1}}{V_{ph2}}$

advantages:

- The available neutral point on primary side can be earthed to avoid distortion.
- It is possible to handle large, unbalanced load.

Disadvantage

- Since the secondary voltage is not in phase with the primary, it is not possible to make it parallel with star-star and delta-delta transformer

Delta-star connection:-

- This type of transformer connection is used to step up the voltages and hence it is used at the beginning of transmission or at the sending end.

For star connection $V_{ph1} = \frac{V_{L1}}{\sqrt{3}}$

For delta connection $V_{L2} = V_{ph2}$

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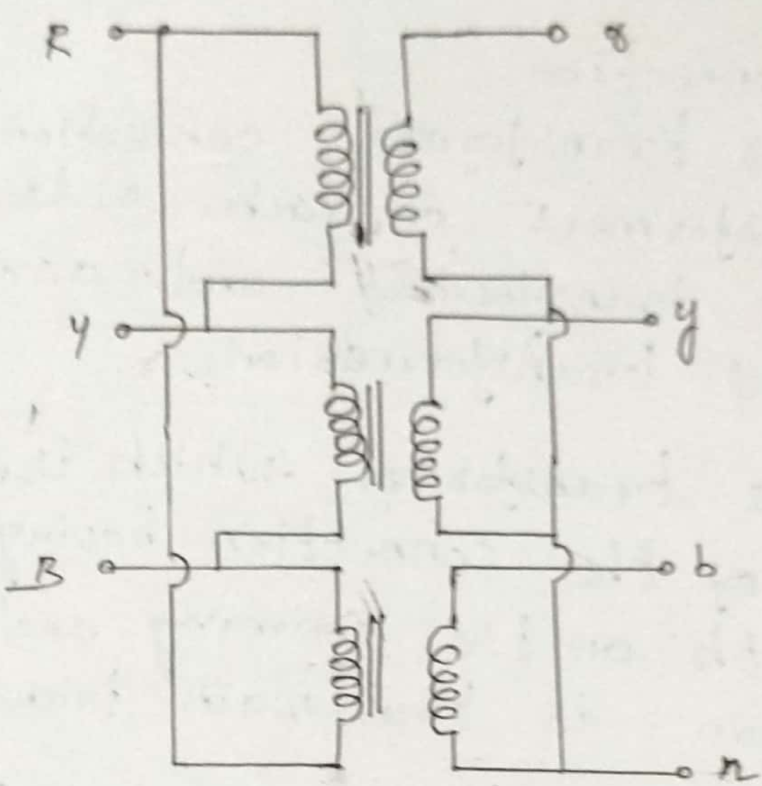
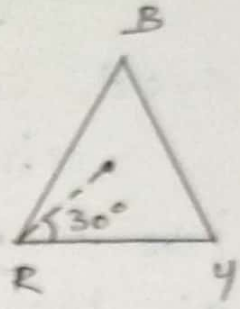
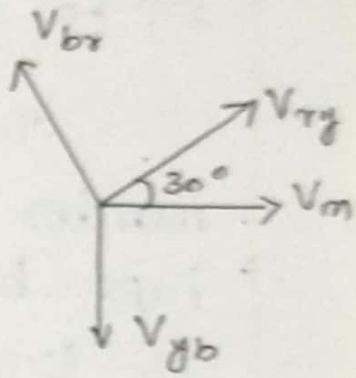
$$V_{ph2} = \frac{1}{a} V_{ph1} = \frac{V_{L1}}{a\sqrt{3}}$$







where $a = \frac{V_{ph1}}{V_{ph2}}$

$$V_{L2} = \sqrt{3}\, V_{ph2} = \sqrt{3}\left(\frac{V_{L1}}{\sqrt{3}}\right)$$

Advantages:

- since neutral is available on the secondary side, 3φ four wire supply can be carried out.
- No distortion occurs due to third harmonic component.

Disadvantage.

- It is not possible to make it parallel with star-star and delta-delta transformers
- It is affected by unbalanced load.

Scott Connection (or) T-T connection.

Fig shows connection diagram of scott connection.

This transformer connection requires two transformers on each side instead of three transformers and accomplishes 3φ to 3φ transformation.

The transformer which is a horizontal member of the connection having centre taps both on the primary and the secondary is known as the main transformer.

The other transformer of primary and secondary whose one end is connected to the main transformer has a 0.866 tap and it is called the teaser transformer.